SCALE: Source Code Analyzer for Locating Errors

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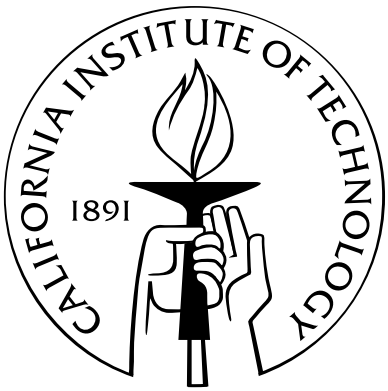

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To my family and friends.

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Abstract

This thesis presents the design and implementation of SCALE, a tool for systematic software testing multi-threaded C applications that use the pthread library. SCALE exhaustively explores the non determinism introduced by thread schedulings and tries to find violations of safety properties. We have designed SCALE to be flexible so that it is easy to add and combine different exploration and state space reduction algorithms. In this thesis we describe the currently implemented reduction algorithms, of which the most important ones are local execution cycle detection and super step partial order reduction. To exemplify how SCALE can be used, we have applied it to a few multi-threaded applications, measured its performance and compared the results to those obtained by other tools. While checking the implementation of a non-blocking queuing algorithm, we were able to find a previously unknown bug that appears only in some unexpected thread inter-leavings.

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Chapter 1

Introduction

Error free multi-threaded applications are hard to write. Concurrency related errors, might be revealed only in unexpected and improbable thread inter-leavings. Even if run for a large amount of time, traditional testing might still miss such errors because it cannot control how the operating system schedules the threads. When an error is found, it cannot be analyzed because there is no way to reliably reproduce it. Model checking [6] is one way to address these problems.

A model checker takes as input a finite state model of a system and a specification, and by exhaustively exploring the state space of the model, it verifies whether it meets the specification or not. As all states are visited, an error state cannot be missed. If the specification can be violated then the model checker generates a counter example that reveals the error. By replaying the counter example, the bug can be reproduced and analyzed.

The size of an application's state space grows rapidly with the number and the size of its concurrent components. This is commonly known as the state space explosion problem. Without some form of state space reduction, even for relatively small systems, model checking is infeasible. Partial order reduction [24, 9, 21] is a technique that was successfully used to reduce the number of states explored. Also, by abstracting the system, its state space can be further reduced. Usually the abstracted system and the original will not have exactly the same properties, but some errors might show up in both. When an error not present in the original system is found in the abstracted system, the abstraction can be refined [2] to account for the difference. Yet another approach [4] is to implicitly represent the state graph as a boolean formula in propositional logic.

Unlike testing, model checkers usually work on a finite state abstract model of the application, and can verify both safety and liveness properties. The abstract model can hide some of the bugs, and it can introduce false positives, errors that can happen in the abstract model but cannot happen in the actual application. Also, every time the application's source code changes, the abstracted model has to be updated.

Systematic software testing is a hybrid approach which, like testing, checks the actual application, but also, like model checking, exhaustively explores the non determinism introduced by thread

schedulings. Much like model checking, systematic software testing suffers from the state space explosion problem: the number of different thread schedulings that have to be tried increases very rapidly with the number and the size of the concurrent components. One drawback of this approach is that it usually does not check liveness properties, it only checks for common safety violations like absence of deadlock, race conditions, and assertion violations.

In this thesis we describe the design of the SCALE (Source Code Analyzer for Locating Errors), a tool for systematic software testing multi-threaded Unix/Linux applications written in C using the pthread library. By exhaustively exploring the non determinism introduced by thread interleavings, it tries to find and report common concurrency errors such as deadlocks, and assertion failures. SCALE can also be used to check safety properties specified as invariants on the global state of the application. We address the state space explosion problem by chaining various state space reduction algorithms. This thesis describes in detail two of the algorithms: local execution cycle detection and super step partial order reduction.

As SCALE checks the actual application, not an abstracted model of the application, it ensures that no spurious violations are reported. All errors that are found are real errors that can manifest when the application is run. When SCALE detects a property violation, it generates a counter example that can later be replayed to reproduce the bug.

We have successfully applied SCALE to a few multi-threaded applications, and we were able to find concurrency related errors. For example we have found a bug in a non-blocking queuing algorithm. Also, by applying our tool to correct systems, we were able to measure the amount of reduction achieved using the local execution cycle detection and super step partial order reduction algorithms. We compare our results to those obtained by other tools and we show that, in some cases, SCALE requires fewer runs than other tools to verify the same applications.

This thesis is organized as follows: In Chapter 2 we briefly present related work in software model checking and systematic software testing. Chapter 3 introduces the multi-threaded model assumed by SCALE. In Chapter 4 we present the state space exploration and state space reduction algorithms implemented in our tool. Chapter 5 presents the design and the implementation of SCALE. Chapter 6 shows some preliminary results obtained by applying our tool to some multi-threaded applications. Finally, in Chapter 7 we conclude and present possible future directions.

Chapter 2

Related Work

A wide variety of model checking and systematic software testing tools that work on C programs have been implemented. Usually, model checkers can verify a larger class of properties specified in temporal logics. Systematic software testing tools only check for safety properties like absence of deadlock, race conditions, and assertion failures.

Another difference between the various tools is how they represent the explored state space. Some tools explicitly store application states. Others store the state space implicitly as a set of constraints. There are even tools that do not store states at all.

Berkeley Lazy Abstraction Software Verification Tool (BLAST)[2] is a model checker for C programs that does counterexample driven automatic abstraction refinement. It constructs an abstract model of the program, which is then model checked for safety properties. If spurious errors are detected, the abstract model is refined and checked again.

CHESS[19] is the first tool to introduce context switch bounding. It verifies multi-threaded C, C++ and .NET code written as test cases. A state is encoded as the happens-before graph of the partial order equivalent executions that lead to that state, thus achieving some partial order reduction. Backtracking to a previous visited state is done by cleaning up the resources used by the test case, restarting it and following a path that leads to the desired state. CHESS checks safety properties like deadlock and data races, and can also find some non progress cycles. CHESS has been successfully used to verify Windows device drivers and also the kernel of a new operating system.

C Model Checker (CMC)[16] has been successfully used to verify protocol code written in C. CMC stores states explicitly and uses heuristics to prioritize the state space search. It is also able to save and restore previously visited states. CMC can check for memory leaks, assertion failures, and global invariants.

Inspect[25] is a systematic software testing tool for checking multi-threaded programs written in C or C++ using the pthread library. It performs a static analysis to identify shared data and then it instruments the program to enable the verification. During state space exploration, Inspect doesn't store application states, but it uses dynamic partial order reduction [7] to reduce the number

of application runs it has to try. Inspect can find common errors like deadlocks, data races and assertion violations.

pan_cam[27] is an explicit state model checker that verifies C programs compiled down to LLVM bytecode [15]. It uses SPIN as a backend to drive the state space exploration. pan_cam uses super step partial order reduction and context switch bounding to reduce the number of states explored. Taking advantage of the SPIN backend, pan_cam can verify both safety and liveness properties expressed as Promela never claims or LTL[22] formulas.

SPIN[11] is an explicit state model checker generator which can be used to verify models written in Promela (Process Meta Language). Since version 4.0 it allows C code to be embedded in the Promela model. Spin can be used to verify both safety and liveness properties. The properties are specified using never claims which represent Buchi automata. Spin can also convert an LTL formula to the corresponding never claim. A wide range of techniques are used to reduce the state space and the memory occupied by the states: partial order reduction, state compression and bitstate hashing.

VeriSoft[10] is a systematic software testing tool that verifies C code. It performs a stateless search and uses partial order reduction to reduce the number of times it has to run the application. VeriSoft can be used to check for assertion violations and coordination problems like deadlocks.

Chapter 3

Multi-Threaded Applications

3.1 Dynamic Semantics

A *Multi-Threaded Application* is a finite set P of threads. We define the state space of the multi-threaded application using a *dynamic semantics* similar to the ones introduced in [10, 7].

Each thread executes a *sequence of statements* described by a *deterministic sequential piece of C code*. The code is deterministic in the sense that presented with the same data, it will execute the same sequence of statements.

Initially, the application has only one *active* thread, called the *main thread*. The main thread executes the sequence of statements found in the main function of the application's C source code. For another thread to become *active*, it must first be *created* by executing a "create" statement from an already active thread. A newly created thread p executes the sequence of statements found in the start routine that was specified in the statement that created p.

An active thread that has ended and has no more statements to execute becomes a *dead* thread. Dead threads cannot be created again, and cannot become active ever again. P, the set of all threads, is finite and ensures that only a finite number of threads are ever created.

Only active threads can execute statements.

Threads communicate with each other by accessing, in their statements, a finite set of *communication variables*. Each access to a communication variable is atomic. We distinguish two classes of communication variables: (a) *data variables* such as global variables and the heap memory, and (b) *synchronization variables* such as threads, mutexes, and condition variables.

A statement that accesses a communication variable or is an assertion is called a *global statement*. Any other statement is called a *local statement*.

At any time the multi-threaded application is said to be in a *state* called the *current state*.

The execution of a statement in the current state is said to *block* if it cannot currently be completed. The execution of a local statement is not allowed to block.

The application is in a *global state* if for every active thread, the next statement to be executed

is a global statement. A *thread exit* is considered to be a global statement, and this ensures that each active thread has at least one global statement. We assume that there are no infinite sequences of local statements, so each thread, after becoming active, eventually reaches a global statement.

The *initial global state* is the state reached when the main thread reaches the first global statement. The creation of a new thread is a global statement, so the main thread is the only active thread in the initial global state. Since the code for the main thread is deterministic and no global statements have been executed before the initial global state is reached, the initial global state is unique and from now on we refer to it as s_0.

A *thread transition* t moves the application from one global state s to another global state s' by executing statements from a single thread: a global statement followed by a *finite* sequence of local statements up to (but not including) the next global statement. Formally, we write $s \xrightarrow{t} s'$. Let T be the set of all transitions.

A transition is *disabled* in a global state s if the execution of its global statement in s would block. A transition that is not disabled in a global state s is said to be *enabled* in s. Note that the execution of an enabled transition always terminates since the global statement won't block, the number of local statements is finite, and each local statement can't block.

A global state s' is said to be *reachable* from a global state s if there exists a finite sequence of transitions w which, taken in order, lead the multi-threaded application from s to s'. Formally, we write $s \stackrel{w}{\Rightarrow} s'$.

A multi-threaded application is a *closed system* evolving from its initial global state s_0 by executing enabled transitions from T. The environment, if any, is either included in the application, or it doesn't store any state. The execution terminates when it reaches a global state in which no transitions are enabled. If the execution terminates in a state s in which there are still active threads, then s is considered to be a *deadlock state*.

The above observations impose the following restrictions on the types of C programs that can be verified by SCALE:

- each thread runs a deterministic sequence of C statements;
- there are no infinite loops that contain only local statements;
- there are no other blocking statements besides the pthread operations;
- if during execution, the application reaches a state s, with the environment being in a state e, then any execution that leads to state s also ensures that the environment is in state e.

SCALE assumes that the application satisfies the above restrictions and has no mechanism to check or enforce them.

Definition 1. A multi-threaded application is represented by a transition system $A_G = (S, \rightarrow, s_0)$ where:

- S is the set of *reachable global states* of the multi-threaded application;
- $\rightarrow \in S \times S$ is the *transition relation*, $(s, s') \in \rightarrow$ if and only if $\exists t \in T. s \xrightarrow{t} s'$;
- s_0 is the *initial global state* of the multi-threaded application.

Let us fix a multi-threaded application A_G for the rest of this chapter.

Definition 2. A *safety property* is considered to be one of the following:

- *absence of deadlock*, which is violated when the multi-threaded application reaches a deadlock state;
- *assertion*, which is violated when the assertion fails;
- *global invariant*, which is a boolean function on the communication variables, and it is violated when the function evaluates to *false*.

Theorem 1. *If there exists a reachable state of the multi-threaded application that violates a safety property, then there exists a global state in A_G where the same safety property is violated.*

Proof. Let s be a reachable state where a safety property is violated.

If s is a deadlock state, then for each thread, the next statement is a global statement because only global statements can block. So s must be a global state in A_G.

If s is an assertion violation, then let p_i be the active thread whose next statement is the assertion. Assertions are global statements, so p_i's next statement is a global statement. If for every other active thread p_j, $j \neq i$, we execute all the statements up to its next global statement, then we end up in a global state s' in A_G. Note that such a global statement must exist for every active thread because the last statement of a thread must be a thread exit which is a global statement. The assertion violation in thread p_i is not affected by the execution of local statements in other threads so the assertion is still violated in s'.

If s is a state in which an invariant is violated, then by executing the local statements in every thread up to the next global statement we end up in a global state s' in A_G. Local statements cannot change communication variables, and invariants only refer to communication variables, so the invariant is still violated in s'. □

Considering the above result, from now on, when we use the term *state* we mean a global state in A_G.

3.2 State Representation

Definition 3. States are elements from the set $State$ where:
$State = SharedState \times LocalStates$, and
$LocalStates = P \rightarrow LocalState^{?}$, where $LocalState^{?} = LocalState \cup \{\perp\}$.

For a state $s = (g, ls)$
$g \in SharedState$ contains the values for all communication variables.
$ls \in LocalStates$ is a function that takes a thread as an argument and returns its local state. If p is an active thread then $ls(p)$ contains the values for all the local variables of p, and also, the values of all the arguments passed to the functions that are currently on p's call stack. If p is not an active thread (either it was not created, or it is dead) then $ls(p) = \perp$.

Definition 4. A transition from a thread p in a local state l is a function:
$t_{p,l} : SharedState \rightarrow LocalState^{?} \times SharedState \times (P \times LocalState)^{?}$.

If in a state $s = (g, ls)$ we have an enabled transition $t_{p,l}$, with $t_{p,l}(g) = (l', g', n)$, then the state $s' = (g', ls')$ reached by taking the transition $t_{p,l}$ in s is obtained as follows:

- g' is the shared state returned by the transition.
- If $l' = \perp$, then thread p has just exited and now $ls'(p) = \perp$. In this situation we must have $n = \perp$ and g' must contain, in the corresponding communication variable, the fact that thread p has ended.
- If $l' \neq \perp$ and $n = \perp$, then $ls'(p) = l'$. The local state of thread p has changed.
- If $l' \neq \perp$ and $n = (p', l'')$, then $ls'(p) = l'$, $ls'(p') = l''$ and g' must contain, in the corresponding communication variable, the fact that thread p' was created.
- If $n = \perp$, then for every thread $i \in P, i \neq p$ we have $ls'(i) = ls(i)$.
- If $n = (p', l'')$, then for every thread $i \in P$, $i \neq p \wedge i \neq p'$, we have $ls'(i) = ls(i)$. A transition from one thread cannot affect the local states of other threads, except the local state of a newly created thread.

To represent a state $s = (g, ls)$ we would like to store the values of all communication variables and also the values of all local variables of each thread. We consider that such an approach is not feasible because it uses too much much memory per stored state. Instead we have opted for a state representation similar to the one introduced in [18, 19].

Definition 5. Two transitions t_1 and t_2, in a state s, are *independent* if:

1. they are both enabled and executing one cannot disable the other one,

2. both the execution of the sequence $t_1;t_2$, and the execution of the sequence $t_2;t_1$ from s lead the multi-threaded application to the same state s', and

3. neither t_1 nor t_2 are assertions or change the truth value of any global invariant.

Two transitions that are not independent are *dependent*. Let $D \subseteq T \times T$ be the dependency relation between transitions.

A similar definition of independence was originally introduced in [13].

Definition 6. Two sequences of transitions are *partial order equivalent* with respect to an independence relation, if they can be obtained from each other by successively exchanging adjacent independent transitions.

Lemma 1. *Let w be a finite sequence of transitions, and let w' be another sequence of transitions obtained from w by exchanging two adjacent independent transitions. If $s \stackrel{w}{\Rightarrow} s'$, then $s \stackrel{w'}{\Rightarrow} s'$.*

Proof. Let $w = t_1t_2...t_it_{i+1}...t_n$, and let $w' = t_1t_2...t_{i+1}t_i...t_n$ be obtained by swapping the adjacent independent transitions t_i and t_{i+1}.

We split w in the following three parts: $w_1 = t_1...t_{i-1}$, $w_2 = t_it_{i+1}$ and $w_3 = t_{i+2}...t_n$.

We also split w' in the following three parts: $w'_1 = t_1...t_{i-1}$, $w'_2 = t_{i+1}t_i$ and $w'_3 = t_{i+2}...t_n$.

Note that $w_1 = w'_1$ and $w_3 = w'_3$.

Let $s \stackrel{w_1}{\Rightarrow} s''$, we also have $s \stackrel{w'_1}{\Rightarrow} s''$.

t_i and t_{i+1} are independent so we must have $s'' \stackrel{w_2}{\Rightarrow} s'''$ and $s'' \stackrel{w'_2}{\Rightarrow} s'''$.

We know that $s \stackrel{w}{\Rightarrow} s'$ and we have $s \stackrel{w_1}{\Rightarrow} s'' \stackrel{w_2}{\Rightarrow} s'''$ so we must have $s''' \stackrel{w_3}{\Rightarrow} s'$.

Because $w_3 = w'_3$ we also have $s''' \stackrel{w'_3}{\Rightarrow} s'$.

Combining the parts we have: $s \stackrel{w'_1}{\Rightarrow} s'' \stackrel{w'_2}{\Rightarrow} s''' \stackrel{w'_3}{\Rightarrow} s'$, so $s \stackrel{w'}{\Rightarrow} s'$. □

Lemma 2. *If w and w' are two partial order equivalent sequences of transitions and $s \stackrel{w}{\Rightarrow} s'$, then $s \stackrel{w'}{\Rightarrow} s'$.*

Proof. From the definition of partial order equivalence we know there exists a finite sequence of sequences of transitions: $<w = w_1, w_2, ..., w_n = w'>$ such that for each $0 < i < n$, w_{i+1} is obtained from w_i by exchanging two independent transitions.

We repeatedly apply Lemma 1 for each pair (w_i, w_{i+1}), starting with (w_1, w_2), and at the end we get that $s \stackrel{w'}{\Rightarrow} s'$. □

Definition 7. A *happens-before graph*[14], corresponding to a finite sequence of transitions $w = t_1t_2...t_n$, starting from s_0, and a dependency relation $D \subseteq T \times T$, is a graph $HB(w) = (A, H)$ where:

- $A = \{t_i \mid 0 < i \leq n\}$ is the set of nodes, one node per each transition in w. If it happens that the same transition appears twice in w, then we have two nodes for it, and we distinguish between them considering the order in which they appear.

- $H = \{(t_i, t_j) \mid 0 < i < j \leq n \wedge (t_i, t_j) \in D^+\}$, the set of edges, is the transitive closure of the dependency relation D on the set A.

Lemma 3. *Two sequences of transitions w_1 and w_2, starting from s_0, are partial order equivalent if and only if $HB(w_1) = HB(w_2)$.*

Proof. Let $HB(w_1) = (A_1, H_1)$ and $HB(w_2) = (A_2, H_2)$. If w_1 is partial order equivalent to w_2, then they must contain the same transitions so we must have $A_1 = A_2$.

For any two independent transitions t_i and t_{i+1} from w_1, we have that $(t_i, t_{i+1}) \notin H_1$. If we swap them and form a new sequence w_1', with $HB(w_1') = (A_1', H_1')$, then $H_1' = H_1$.

w_1 and w_2 are partial order equivalent, so there exists a sequence of sequences of transitions $<w_1 = v_1, v_2, ..., v_n = w_2>$ such that for each $0 < i < n$, v_{i+1} is obtained from v_i by exchanging two independent transitions. Repeatedly applying the previous argument for adjacent pairs (v_i, v_{i+1}), $0 < i < n$, we get that $H_1 = H_2$.

So we must have $HB(w_1) = HB(w_2)$.

We prove by induction on the length of w_1 that if $HB(w_1) = HB(w_2)$ then w_1 is partial order equivalent to w_2.

If $HB(w_1) = HB(w_2)$, then $A_1 = A_2$, and so w_1 and w_2 contain the same transitions, maybe in a different order.

For a w_1 of length 0, w_2 must also have length 0 and so they are partial order equivalent.

For a $w_1 = t_1$ of length 1, we must also have $w_2 = t_1$ since they have the same transitions.

Assuming that for any w_1' and w_2', of length $n - 1, n > 0$, if $HB(w_1') = HB(w_2')$, then w_1' and w_2' are partial order equivalent, we proceed to prove that for any w_1 and w_2 of length n, if $HB(w_1) = HB(w_2)$ then w_1 and w_2 are partial order equivalent.

We have two cases:

(a) $w_1 = t_1...t_n$ and $w_2 = t_1'...t_n'$ with $t_n = t_n'$. We can remove the node that corresponds to t_n from both of $HB(w_1)$ and $HB(w_2)$ and we are left with the happens before graphs for $t_1...t_{n-1}$ and $t_1'...t_{n-1}'$. These graphs must be identical (because we've removed the same node from both) and so we can apply the induction hypothesis and get that $t_1...t_{n-1}$ is partial order equivalent to $t_1'...t_{n-1}'$.

Since $t_n = t_n'$ we can conclude that w_1 and w_2 are partial order equivalent.

(b) $w_1 = t_1...t_n$ and $w_2 = t_1'...t_n'$ with $t_n \neq t_n'$.

Because w_1 and w_2 have the same transitions, there exists a transition $t_i' = t_n$. t_n is the last transition in w_1, meaning that it cannot have outgoing edges in H_1. Since $H_1 = H_2$, for any $t_j', j > i$ we have $(t_i', t_j') \notin H_2 \wedge (t_j', t_i') \notin H_2$. In other words, t_i' is independent with all the transitions that occur in w_2 from its place up to the end of w_2. We can repeatedly swap t_i' with these transitions until t_i' becomes the last transition.

We have constructed a new sequence, w_2', which is partial order equivalent to w_2. Using the first

part of this proof, we know that $HB(w_2') = HB(w_2)$, so $HB(w_2') = HB(w_1)$, and moreover, w_2' and w_1 have the same last transition. We can use case (a) to prove that w_1 and w_2' are partial order equivalent, so w_1 and w_2 are partial order equivalent. □

Definition 8. For a state s and a finite sequence of transitions w such that $s_0 \stackrel{w}{\Rightarrow} s$, we say that $HB(w)$ is a *representation* of s.

Note that only reachable states can be represented. Also, there might be more than one representation for a single state, corresponding to different transition sequences that lead to it.

Theorem 2. *Let's consider a state s_1 that has a representation (A_1, H_1) and a state s_2 that has a representation (A_2, H_2). If $(A_1, H_1) = (A_2, H_2)$, then $s_1 = s_2$.*

Proof. s_1 has a representation implies that there exists a sequence of transitions w_1 such that $s_0 \stackrel{w_1}{\Rightarrow} s_1$ and $(A_1, H_1) = HB(w_1)$.

s_2 has a representation implies that there exists a sequence of transitions w_2 such that $s_0 \stackrel{w_2}{\Rightarrow} s_2$ and $(A_2, H_2) = HB(w_2)$.

We have $HB(w_1) = HB(w_2)$ which implies, using Lemma 3, that w_1 and w_2 are partial order equivalent.

From Lemma 2 we know that two partial order equivalent sequences of transitions, if executed from the same state s_0, lead to the same state s, so we must have $s_1 = s_2$. □

Theorem 3. *Each state s from A_G has at least one representation.*

Proof. s is in A_G, and A_G contains only the reachable states, so there must exist a w such that $s_0 \stackrel{w}{\Rightarrow} s$.

Then $HB(w)$ is a representation of s. □

Theorem 4. *If A_G is acyclic, then each state s from A_G has only a finite number of representations.*

Proof. From Graph Theory we know that if a graph is acyclic then it contains only a finite number of different paths.

For our case, there are a finite number of paths from s_0 to s, so there are a finite number of transition sequences w such that $s_0 \stackrel{w}{\Rightarrow} s$. Each such transition sequence w can generate at most one different representation $HB(w)$ for s.

So s can only have a finite number of representations. □

In all the previous arguments we've assumed that we know the transitions and that we can store them. For a C program this would not be feasible. For example, a transition might involve some library calls, and it would be complicated to come up with a function that represents what the call does and compose it with all the other statements in the same transition.

Luckily, if we take a closer look at our arguments, the only thing that we have assumed is that we can identify which thread takes each transition, which transitions are enabled in the current state, and which transitions are dependent. Our definition of the state space A_G requires us to also be able to apply a transition t to a state s and get the new state s'. We can do that by running the actual application. Assuming that the application is in the state s, we can schedule the thread that corresponds to transition t and let it run up to the next global statement. The application is guaranteed now to be in state s' because we have assumed that all transitions are deterministic.

Whenever we want to get to a certain state s, reachable through a sequence of transitions w, we can restart the application from its initial state s_0 and schedule, in order, the threads that correspond to the transitions in w. Because all the transitions are deterministic, we are guaranteed to end up in s. So what we only need in order to recreate s, is the sequence of threads that have to be scheduled from s_0. The same sequence of threads will always lead, when scheduled from s_0, to the same state s.

We also have to identify which transitions create new threads, and which transitions exit active threads.

We can over-approximate the dependency relation as follows.

Definition 9. Two transitions are *dependent* if any of the following is true:

- they access the same communication variable, and at least one of the transitions writes to the variable;
- at least one of the transitions is an assertion;
- at least one of the transitions accesses a communication variable used to calculate the value of at least one global invariant.

Any access to a synchronization variable is considered to be a write access.

We abstract all the information we need to know about a transition in an action.

Definition 10. For a transition t, the corresponding *action* is a triple (p, op, vl) where:

- p is the thread taking the transition t;
- op is the operation performed in the transition t. For example, it can be a *read*, a *write*, a $thread\_create$, a $thread\_exit$, $mutex\_lock$, etc.;
- vl is a list of communication variables accessed by the transition t. For example, if the transition creates a new thread, then the variable list specifies which thread is created, if the transition reads a variable, then the variable list specifies which variable is read, etc.

An action a, which abstracts a transition t, is *enabled* in a state s if t is enabled in s. An action that is not enabled is *disabled*.

We can extend the dependency relation and the partial order equivalence, Definitions 9 and 6, from transitions to actions.

If we have $s_0 \stackrel{w}{\Rightarrow} s$, then we can use a scheduling mechanism and the sequence of actions that corresponds to w to recreate s. Also, every time we schedule the same sequence of actions from s_0, we are guaranteed to reach the same state state s.

From what we have said before, we know that we could use a sequence of threads, or a sequence of actions, to represent a state s that is reachable from s_0. But, by doing so, we would get different representations of s even for executions that are partial order equivalent. As it was the case with transitions, we can use a happens-before graph to create a single representation for all partial order equivalent executions.

Definition 11. A *happens-before graph for actions*, corresponding to a finite sequence of actions $q = a_1 a_2 ... a_n$, is a graph $HBA(q) = (A, H)$ where:

- $A = \{a_i \mid 0 < i \leq n\}$ is the set of nodes, one node for each action. If the same action appears multiple times in the sequence we create a new node for each time it appears and we distinguish between them considering their order in the sequence.
- $H = \{(a_i, a_j) \mid 0 < i < j \leq n \wedge (a_i, a_j) \in D^+\}$, where D^+ is the transitive closure of the dependency relation D for actions from Definition 9.

Definition 11 is similar to Definition 7, but it uses actions instead of transitions.

From now on, to represent a state s, reachable from s_0 by scheduling a sequence q of actions, we will use the happens-before graph for actions $HBA(q)$.

Lemmas 1, 2, 3, and Theorems 2, 3, 4 also hold for sequences of actions and happens-before graphs for actions. The reason why this is true is that using actions and a scheduling mechanism, we can reconstruct both the effect of transitions and the dependency relation between transitions, assuming that transitions are deterministic.

Before we can present the state space exploration algorithms, we have to make some assumptions about the scheduling mechanism. In Chapter 5 we will show how such a mechanism can be implemented.

Remember that a state s was a tuple (g, ls) where g is the shared state and ls is the local state for each thread. We assume that g is a tuple (dv, sv) where dv holds the values for all the data variables (global variables and the heap), and sv holds the values for all the synchronization variables. Considering the memory layout of an application, presented in Figure 3.1, we can identify the components of the state. For each thread, the local state is stored in the thread's stack. The

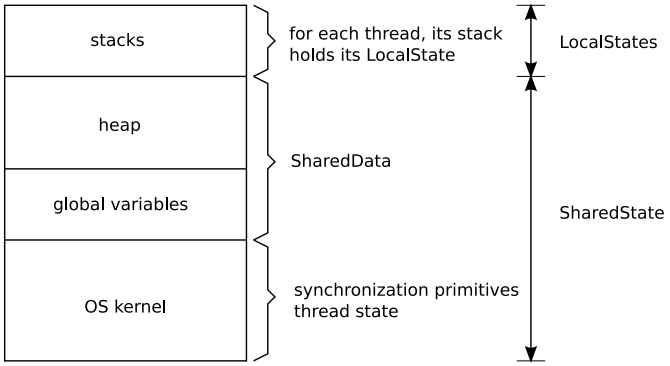


Figure 3.1: The memory layout of an application

data variables are stored in the global variables and the heap. The synchronization variables are stored in the OS kernel.

We assume that the scheduling mechanism gives us read access to the stacks of each thread and also to the data variables that correspond to the current state of the application. The scheduling mechanism also allows us to evaluate invariants in the current state.

We assume that the scheduling mechanism allows us to get the application to its initial state s_0, and from that state, allows us to schedule one thread at a time. For each thread scheduling p, the mechanism atomically executes the transition t that corresponds to thread p, and the current state of the application becomes s', assuming that we had $s \xrightarrow{t} s'$.

We assume that the scheduling mechanism allows us to get what is the configuration corresponding to the current state.

Definition 12. A *configuration* that corresponds to a state s is a set of tuples $(a, enabled, new)$, one tuple for each active thread p, where:

- a is the next action of thread p in state s;
- *enabled* is *true* if a is enabled in s, and *false* otherwise.
- *new* is *true* if a is from a thread p that was just created, and *false* otherwise. Only one tuple can have $new = true$.

Definition 13. We make the following notations for a configuration $config$:

- $actions(config)$ is the set of all available next actions including both the enabled and the disabled ones;
- $enabled\_actions(config)$ is the set of enabled next actions;
- $enabled\_threads(config)$ is the set of threads that have an enabled next action;

- $new\_action(config)$ is $\perp$ if no new thread was created, or the next action that corresponds to the new thread;
- $new\_thread(config)$ is $\perp$ if no new thread was created, or the new thread if one was created.

Definition 14. For a happens-before graph (A, H) that represents a state s, and an action a' enabled in s, $extend((A, H), a')$ is a happens-before graph (A', H') with:

- $A' = A \cup \{a'\}$. Add a new node for the action a';
- $H' = H \cup \{(a, a') \mid a \in A \wedge (a, a') \in D^+\}$, is the transitive closure of the dependency relation D on A'.

Lemma 4. *If (A, H) is a representation of s, a' is an action enabled in s that abstracts a transition t', and $s \xrightarrow{t'} s'$, then $extend((A, H), a')$ is a representation of s'.*

Proof. Let $extend((A, H), a') = (A', H')$.

(A, H) is a representation of s implies that there exists a sequence of actions $q = a_1...a_n$ that abstracts a sequence of transitions $w = t_1...t_n$ such that $s_0 \stackrel{w}{\Rightarrow} s$.

We construct a new sequence of transitions $w' = t_1...t_nt_{n+1}$, with $t_{n+1} = t'$, and from $s \xrightarrow{t} s'$ we have that $s_0 \stackrel{w'}{\Rightarrow} s'$. Let $q' = a_1...a_na_{n+1}$, with $a_{n+1} = a'$, the sequence of actions that abstracts w'.

$HBA(q') = (A'', H'')$ is a representation for s'.

What is left to show is that $(A'', H'') = (A', H')$.

From Definition 11 we have $A'' = \{a_i | 0 < i \leq n + 1\} = \{a_i | 0 < i \leq n\} \cup \{a'\} = A \cup \{a'\}$ which is the same as A' in Definition 14. So $A'' = A'$.

Also, from Definition 11 we have $H'' = \{(a_i, a_j) \mid 0 < i < j \leq n + 1 \wedge (a_i, a_j) \in D^+\} = \{(a_i, a_j) \mid 0 < i < j \leq n \wedge (a_i, a_j) \in D^+\} \cup \{(a_i, a_{n+1}) \mid 0 < i \leq n \wedge (a_i, a_{n+1} \in D^+\} = H \cup \{(a, a') \mid a \in A \wedge (a, a') \in D^+\}$ which is the same as H'. So $H'' = H'$.

We have $extend((A, H), a') = (A'', H'')$, so $extend((a, H), a')$ is a representation of s'. □

Definition 15. The depth of a happens before graph for actions (A, H) is defined as the number of nodes of the graph: $depth((A, H)) = |A|$.

Lemma 5. *Given a happens before graph (A, H) and an action a, $depth(extend((A, H), a)) = depth((A, H)) + 1$*

Proof. Let $(A', H') = extend((A, H), a)$. From Definition 14 we get that $|A'| = |A| + 1$, so $depth(extend((A, H), a)) = depth((A, H)) + 1$. □

Chapter 4

State Space Exploration

4.1 Exploration Algorithms

Definition 16. For a state representation n that corresponds to a state s, a call of $replay(n)$ ensures that the current state of the application becomes s, and also, the call returns the configuration that corresponds to s.

Definition 17. A state s is considered to be *explored* if, during the exploration, s was at least once the implicit current state of the application.

Lemma 6. *A state s is explored by an exploration algorithm, if the algorithm calls replay at least once with a representation (A, H) of s as an argument.*

Proof. Calling *replay* with a representation of s as an argument ensures that the current state of the application becomes s. Using Definition 17 we can conclude that s is explored. □

If *replay* is the only way the current state can be changed, then the implication is actually an equivalence.

Definition 18. Assuming that $A_G = (S, \rightarrow, s_0)$ is acyclic, $A_R = (V_R, E_R, u_0)$ is a graph of representations in which:

- $V_R = \{HBA(q) \mid \exists s \exists w . s_0 \stackrel{w}{\Rightarrow} s \wedge q = abstract(w)\}$. Any representation of a state from A_G is a node in A_R.
- $E_R = \{(HBA(q), HBA(q; a)) \mid \exists s \exists w \exists t \exists s' . s_0 \stackrel{w}{\Rightarrow} s \wedge s \stackrel{t}{\rightarrow} s' \wedge q = abstract(w) \wedge a = abstract(t)\}$. If there exists a transition between two states s and s' in A_G, then there exists an edge between any representation of s and any representation of s' that extends the representation of s. $HBA(q; a) = extend(HBA(q), a)$.
- $u_o = HBA(\epsilon)$ is a node that corresponds to the initial state.

Lemma 7. *If A_G is acyclic, then the corresponding A_R has a finite number of nodes (A_R is well defined).*

Proof. A_G has a finite number of states.

A_G is acyclic, and using Theorem 4 we know that each state has only a finite number of representations.

The nodes of A_R are all the representations for all the states, we have a finite number of representations for each state, and a finite number of states, so A_R has a finite number of nodes. □

Lemma 8. *A_R is a acyclic.*

Proof. Let's assume that $A_R = (V_R, E_R, u_0)$ contains a cycle: $(v_0, v_1, ..., v_n)$, with $(\forall i.0 \leq i \leq n.v_i \in V_R) \wedge (v_0 = v_n) \wedge (\forall i, 0 \leq i < n.(v_i, v_{i+1}) \in E_R)$. From Definition 18 we know that for any i, $0 \leq i < n$, there exists an action a_{i+1} such that $v_{i+1} = extend(v_i, a_{i+1})$. From Lemma 5 we have that $depth(v_{i+1}) = depth(v_i) + 1$, so $depth(v_i) < depth(v_{i+1})$. Repeatedly applying this observation, we can conclude that $depth(v_0) < depth(v_n)$, but $v_0 = v_n$ and we should have $depth(v_0) = depth(v_n)$. We have reached a contradiction, so our assumption that A_R is not acyclic is wrong. □

Algorithm 1 Basic stateless exploration algorithm.

```
1:  Frontier = {HBA(ε)}
2:  while Frontier ≠ ∅ do
3:     n = pick(Frontier)
4:     Frontier = Frontier \ {n}
5:     config = replay(n)
6:     Successors = ∅
7:     for all a ∈ enabled_actions(config) do
8:        Successors = Successors ∪ {extend(n, a)}
9:     end for
10:    Frontier = Frontier ∪ Successors
11: end while
```

Lemma 9. *If A_G is acyclic, and A_R is the graph of representations that corresponds to A_G, then Algorithm 1 terminates and every node in A_R is eventually picked in line 3 of the algorithm.*

Proof. First we prove that $Frontier$ holds only nodes from A_R. This is an invariant for the while loop.

The invariant holds at the beginning of the loop:

After the execution of line 1, right before the while loop, $Frontier$ contains u_0 which is a node from A_R.

The invariant is maintained by the loop body:

In line 3 of the algorithm we know that $Frontier$ is not empty (from the guard), and assuming the invariant holds at the beginning of the loop, $Frontier$ only contains nodes from A_R.

At the end of line 3, n is an element from $Frontier$ so it is a node from A_R, a representation of a state s in A_G.

In line 4, we remove n from $Frontier$, so it still contains only nodes from A_R, or it is empty.

n is a representation of some state s from A_G, and by calling *replay* on n we ensure that, after line 5 is executed, the implicit current state of the application becomes s and $config$ is the configuration that corresponds to s.

In line 7, $enabled\_actions(config)$ is a set that contains all actions that are enabled in the current state s (Definition 13).

In line 8, n is a representation of s and a is enabled. Using Lemma 4 we get that $extend(n, a)$ is a representation of a state s' that is a successor of s in A_G. From Definition 18 we know that $extend(n, a)$ is a successor of n in A_R. So after line 9, $Successors$ is the set of all successors of n in A_R.

In line 10, $Frontier$ is extended with the set $Successors$, which contains only nodes in A_R. So at the end of line 10, $Frontier$ contains only nodes from A_R.

So far we have proved that $Frontier$ only holds nodes from A_R, n is a node from A_R, and at the end of line 9, $Successors$ contains all the successors of n.

We can now regard Algorithm 1 as a generic graph traversal algorithm that visits nodes from A_R without storing them.

From Graph Theory we know that if A_R is acyclic, then such a traversal terminates and visits all the nodes.

From Lemma 8 we know that A_R is acyclic, so Algorithm 1 will visit every node of A_R, meaning that every node is eventually picked in line 3. □

Lemma 10. *If for every node n in A_R, an exploration algorithm calls $replay(n)$ at least once, then it necessarily explores all states in A_G.*

Proof. From Theorem 3 we know that every state from A_G has a representation. Definition 18 ensures that A_R contains all the representations for all the states in A_G. So for any state s from A_G, A_R contains at least one node n that is a representation of s. The exploration algorithm calls $replay(n)$, and using Lemma 6 we get that s is explored. □

Theorem 5. *If A_G is acyclic, then Algorithm 1 terminates and explores all the states in A_G.*

Proof. By applying Lemma 9 we get that Algorithm 1 terminates.

Lemma 9 ensures that Algorithm 1 calls $replay(n)$ for every node n in A_R. Using Lemma 10 we get that all the states in A_G are explored. □

Lemma 11. *If $Filters = \emptyset$, then Algorithm 2 behaves exactly like Algorithm 1.*

Algorithm 2 Exploration algorithm with state filtering.

```
1:  Frontier = {HBA(ε)}
2:  while Frontier ≠ ∅ do
3:     n = pick(Frontier)
4:     Frontier = Frontier \ {n}
5:     config = replay(n)
6:     Successors = ∅
7:     for all a ∈ enabled_actions(config) do
8:        Successors = Successors ∪ {extend(n, a)}
9:     end for
10:    for all f ∈ Filters do
11:       Successors = f(Successors)
12:    end for
13:    Frontier = Frontier ∪ Successors
14: end while
```

Proof. If $Filters = \emptyset$, then line 11 of Algorithm 2 will never be executed and the set $Successors$, at the end of line 12 will be the same as at the end of line 9. The algorithm has the same effect as if lines 10 - 12 were removed. Removing lines 10 - 12 leaves us with the code for Algorithm 1. □

4.2 State Filters

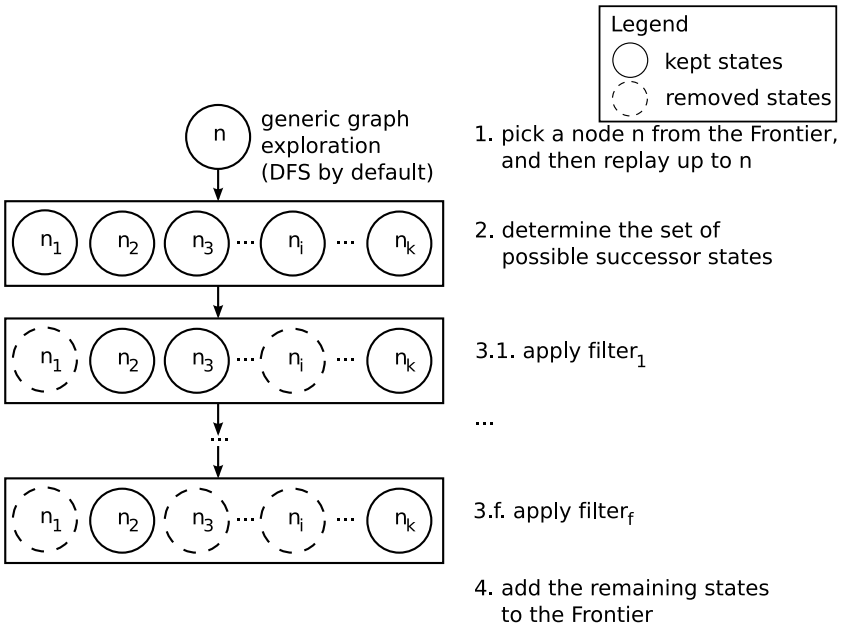


Figure 4.1: Exploration using state filters

Each state filter can tag extra information to each state representation. The extra information can be retrieved in a later call of the filter function. Also, a state filter can add some global

information about all the state representations seen so far.

4.2.1 The $NotSeenBefore$ Filter

Algorithm 3 $NotSeenBeforeFilter(Successors)$

```
1: H = H ∪ {n}
2: for all n' ∈ Successors do
3:    if n' ∈ H then
4:       Successors = Successors \ {n'}
5:    end if
6: end for
7: return Successors
```

The extra information used by the $NotSeenBeforeFilter$ in Algorithm 3 is a set (a cache) of already visited state representations.

Theorem 6. *If A_G is a directed acyclic graph and the set $Filters$ in Algorithm 2 contains only the function presented in Algorithm 3, then Algorithm 2 terminates and explores all the states in A_G.*

Proof. Algorithm 3 is called for each iteration of the while loop in Algorithm 2. In each iteration, Algorithm 3 can only remove elements from the set $Successors$ (line 4).

Line 1 of Algorithm 3 gets called for each visited state representation n, so the set H stores the representations visited so far.

The combination of the two algorithms is a restricted traversal of the graph A_R in which previously visited nodes are not revisited. From Graph Theory we know that such a traversal terminates and visits all the nodes in A_R. So for any node n in A_R, Algorithm 2 calls, in line 5, $replay(n)$. Using Lemma 10 we get that all the states in A_G are explored. □

The main difference between Algorithm 1 and Algorithm 2 with the $NotSeenBefore$ filter is that the first one doesn't store any state representations, while the second one stores all the states representations. It is a trade-off between memory consumption and doing the same work multiple times. The two algorithms are at the extremes of this trade-off. Algorithm 1 uses little memory, but cannot avoid doing extra work when it encounters a node that was previously visited. Algorithm 2 with the $NotSeenBefore$ filter avoids exploring nodes more than once, but it uses a lot more memory because it has to store all visited nodes. By modifying H in Algorithm 3 from a set that stores all the nodes, to a cache that can store only a limited number of nodes, we could control the trade-off between memory consumption and duplicate work.

4.2.2 The $DepthBounding$ Filter

Lemma 12. *If A_G contains at least one cycle, and if we construct A_R according to Definition 18, then A_R has an infinite number of nodes.*

Proof. Let's pick a cycle from A_G: $s_1 \xrightarrow{t_1} s_2 \xrightarrow{t_2} s_3 ... \xrightarrow{t_{n-1}} s_n$, with $n > 1 \wedge s_1 = s_n$.

Let a_i be the action that abstracts transition t_i for all $0 < i < n$.

Let n be a representation of s_1, and let us extend n with a_1, the resulting state with a_2, and so on, until we have extended n with the sequence $a_1 ... a_{n-1}$ and we reach a representation $n^{(1)}$ that is also a representation of s_1 (we have executed the cycle once). $depth(n^{(1)}) = depth(n) + n - 1$, the depth is increase by the size of the cycle.

We can now repeat the argument and extend $n^{(1)}$ to get $n^{(2)}$, another representation for s_1 that has $depth(n^{(2)}) = depth(n) + 2(n-1)$. In fact for any natural number k, we can extend n with the actions in the cycle k times and we get $depth(n^{(k)}) = depth(n) + k(n-1)$.

We have an infinite number of representations for s_1, so A_R has an infinite number of nodes. Note that any state from the cycle has an infinite number of representations, not just s_1. □

Considering the result presented in Lemma 12, the exploration algorithms presented so far (Algorithm 1 and Algorithm 2 with the $NotSeenBefore$ filter), do not terminate if they are run for a multi-threaded application A_G that has a cycle.

The common solution for such a problem is to use a technique called depth bounding[5], the model checker verifies finite executions that are not longer than a certain bound. The usual approach is to set a large depth bound, and whenever this bound is reached, declare that a live-lock was found [10, 19]. Algorithm 4 shows how depth bounding is implemented in SCALE as a state filter.

Algorithm 4 $DepthBoundingFilter(Successors)$

```
1: for all n' ∈ Successors do
2:   if depth(n') > DEPTH_BOUND then
3:     Successors = Successors \ {n'}
4:   end if
5: end for
6: return Successors
```

Theorem 7. *If Algorithm 2 has in the Filters list Algorithms 3 and 4, then it explores all the states from A_G that are reachable through a sequence of transitions of length at most $DEPTH\_BOUND$.*

Proof. Let $A'_R = (V'_R, E'_R)$ be a sub-graph of $A_R = (V_R, E_R)$ such that:

- $V'_R = \{v \in V_R \mid depth(v) <= DEPTH\_BOUND\}$, the nodes that do not exceed the depth bound;
- $E'_R = \{(u, v) \in E_R \mid u \in V'_R \wedge v \in V'_R\}$, the restriction of E_R to the nodes in V'_R.

From the definition of A_R (Definition 18), and the definition of *depth* (Definition 15), we know that if $(u, v) \in E_R$ then $depth(v) = depth(u) + 1$. So for any two nodes $u, v \in V_R$ with $depth(v) < depth(u)$ we have that $(u, v) \notin E_R$. A_R does not contain edges from representations with a higher

depth to representations with a lower depth. So there is no edge in E_R from a node that is in $V_R \setminus V'_R$ to a node that is in V'_R.

The *DepthBounding* filter limits the traversal of A_R to that of A'_R. Applying a similar argument to the one used in Theorem 6, we get that the exploration algorithm visits all the nodes in A'_R.

For any state s from A_G that is reachable through a sequence of transitions w that has a length of at most $DEPTH\_BOUND$, there exists a representation n in A'_R that corresponds to the sequence of actions q that abstracts w. n is visited by the exploration algorithm, so the corresponding state s is explored (because $replay(n)$ is called in line 5 of Algorithm 2). □

4.2.3 The *LocalCycleDetection* Filter

From now on we assume that the implementation of *extend* and the way a happens-before graph for actions is stored allows us to recover, for any state representation, the parent state representation and the action used to extend the parent representation.

Definition 19. For a state representation $n' = extend(n, a)$ we define:

- $parent(n') = n$, the parent state representation from which n' was extended;
- $last\_action(n') = a$, the action used to extend the parent state representation to get the new state representation n';
- $last\_thread(n') = p$, if $a = (p, op, vl)$. p is the thread that executed the last action;

For u_0, the representation of s_0 we define:

- $parent(u_0) = \perp$, u_0 was not obtained by extending an already existent state representation;
- $last\_action(u_0) = \perp$;
- $last\_thread(u_0) = m$, where m is the main thread.

In Chapter 5 we will present how we can efficiently store and extend happens-before graphs, and at the same time preserve the *parent* and *last_action* information.We will also show how we can efficiently hash state representations. Storing the hash of a representation instead of the representation can reduce the memory used by the $NotSeenBefore$ filter, and also decrease the time required to test the equality of two representations. There is a drawback to using hashing: we no longer have a guarantee that all state representations will be explored.

Definition 20. A *local execution cycle* from A_G is a cycle $s_1 \xrightarrow{t_1} s_2 \xrightarrow{t_2} s_3 ... \xrightarrow{t_{n-1}} s_n$, with $s_1 = s_n$ in which all the transitions $t_i, 0 < i < n$ are taken by the same thread t and none of them change a communication variable.

Algorithm 5 $LocalCycleDetection(Successors)$

```
1: A = ∅
2: t = last_thread(n)
3: LocalState(n) = get_stack(t)
4: a = last_action(n)
5: p = parent(n)
6: progress = false
7: while p ≠ ⊥ ∧ ¬progress ∧ Successors ≠ ∅ do
8:    assert a ≠ ⊥
9:    if MadeProgress(a, t) ∨ conflicts(a, A, t) then
10:      progress = true
11:   else
12:      if last_thread(p) = t ∧ LocalState(p) = LocalState(n) then
13:         Successors = ∅
14:      else
15:         A = A ∪ {a}
16:         a = last_action(p)
17:         p = parent(p)
18:      end if
19:   end if
20: end while
21: return Successors
```

Definition 21. For each visited state representation $n \in A_R$, Algorithm 5 tags $LocalState(n)$ as extra information.

Definition 22. In Algorithm 5, $get\_stack(t)$ returns the local state associated with thread t.

Definition 23. In Algorithm 5, for an action a and a thread t:

- $MadeProgress(a, t)$ is *true* if an only if $thread(a) = t$, and the operation corresponding to a changes a communication variable;
- for a set of actions A, $conflicts(a, A, t)$ is *true* if and only if there exists an action $a' \in A$ such that $(a, a') \in D \wedge ((thread(a) = t \wedge thread(a') \neq t) \vee (thread(a) \neq t \wedge thread(a') = t))$, where D is the dependency relation form Definition 9.

Definition 24. A sequence of state representations from A_R, $q = u_1 \xrightarrow{a_1} u_2 ... \xrightarrow{a_k} u_{k+1}$ is said to *exhibit a local execution* from A_G if and only if there exists a sequence $q' = u'_1 \xrightarrow{a'_1} ... \xrightarrow{a'_k} u'_{k+1}$ that is partial order equivalent to q such that:

- $a'_k = a_k$,
- there exists i, $0 < i \leq k$, such that $last\_thread(u'_i) = last\_thread(u_{k+1})$,
- for all l, $0 < l \leq i$, $u'_l = u_l$,
- for all l, $0 < l < i$, $a'_l = a_l$,

- there exists j, $i \leq j \leq k$, such that
  - for all l, $j \leq l \leq k$, $thread(u'_l) = thread(u_{k+1})$,
  - for all l, $i < l < j$, $thread(u'_l) \neq thread(u_{k+1})$,
  - for all l, $i < l < j$, and for all m, $j \leq m \leq k$, $(a_l, a_m) \notin D$,
  - $u'_j \xrightarrow{a'_j} ... \xrightarrow{a'_k} u'_{k+1}$ corresponds to a local execution cycle in A_G.

Lemma 13. *For any node $n \in A_R$ which was obtained by extending the initial node u_0 with a sequence of actions $q = a_1a_2...a_k$, $k \geq 0$, and for any set Successors, Algorithm 5 terminates. Moreover, if q exhibits a local execution cycle starting from u_0 then, at the end of the algorithm, Successors becomes $\emptyset$, otherwise Successors is not changed.*

Proof. In each iteration of the loop, either $depth(p)$ is decreased by 1 or p becomes $\perp$ (line 17), or *progress* is set to *true* (line 10), or *Successors* is set to $\emptyset$ (line 13). If *progress* is set to *true* or *Successors* is set to $\emptyset$, then the loop is exited because of the loop guard. Also, $depth(p)$ cannot decrease forever because it has a lower bound of 0 reached when $p = u_0$. $parent(u_0) = \perp$. So eventually the loop is exited, and Algorithm 5 terminates.

Let us define the property *Cycle* to be true if and only if $q = a_1...a_k$ exhibits a local execution cycle (Definition 24).
Let $Successors^0$ be the value of *Successors* at the beginning of Algorithm 5.
What we have to prove is that at the end of Algorithm 5:
$(Cycle \Rightarrow Successors = \emptyset) \wedge (\neg Cycle \Rightarrow Successors = Successors^0)$
Let's call this property *Post*. We have:
Post
$\equiv$ {definition of *Post*}
$(Cycle \Rightarrow Successors = \emptyset) \wedge (\neg Cycle \Rightarrow Successors = Successors^0)$
$\equiv$ $\{(P \Rightarrow Q) \equiv (\neg P \vee Q)\}$
$(\neg Cycle \vee Successors = \emptyset) \wedge (Cycle \vee Successors = Successors^0)$
$\equiv$ {distributivity of $\wedge$ over $\vee$}
$(\neg Cycle \wedge Cycle) \vee (\neg Cycle \wedge Successors = Successors^0)$
$\vee((Cycle \vee Successors = Successors^0) \wedge Successors = \emptyset)$
$\equiv$ $\{\ P \wedge \neg P \equiv false\ ;\ false \vee P \equiv P\}$
$(\neg Cycle \wedge Successors = Successors^0) \vee ((Cycle \vee Successors = Successors^0) \wedge Successors = \emptyset)$
Assuming that $J \equiv J_1 \wedge J_2$ is a loop invariant, with:
$J_1 \equiv progress \Rightarrow (\neg Cycle \wedge Successors = Successors^0)$, which is the same as
$J_1 \equiv \neg progress \vee (\neg Cycle \wedge Successors = Successors^0)$,
and

$J_2 \equiv Successors = \emptyset \Rightarrow (Cycle \vee Successors = Successors^0)$, which is the same as

$J_2 \equiv Successors \neq \emptyset \vee Cycle \vee Successors = Successors^0$,

we can prove that Pre is satisfied at the end of the while loop.

Let $G \equiv \neg progress \wedge p \neq \perp \wedge Successors \neq \emptyset$ be a notation for the while loop guard.

At the end of the while loop we have $\neg G \wedge J$ and we have to prove that $(\neg G \wedge J) \Rightarrow Post$.

$(\neg G \wedge J) \Rightarrow Post$

$\equiv$ {definitions of G and J, properties of $\neg$ }

$((progress \vee p = \perp \vee Successors = \emptyset) \wedge J_1 \wedge J_2) \Rightarrow Post$

$\equiv$ { distributivity of $\wedge$ over $\vee$; definitions of J_1 and J_2}

$((progress \wedge (\neg progress \vee (\neg Cycle \wedge Successors = Successors^0)) \wedge J_2) \vee (p = \perp \wedge J_1 \wedge J_2) \vee$

$((Successors = \emptyset) \wedge J_1 \wedge (Successors \neq \emptyset \vee Cycle \vee Successors = Successors^0))) \Rightarrow Post$

$\equiv$ {$P \wedge \neg P \equiv false$; definition of $Post$}

$((progress \wedge (\neg Cycle \wedge Successors = Successors^0) \wedge J_2) \vee ((Successors = \emptyset) \wedge J_1 \wedge (Cycle \vee$

$Successors = Successors^0)) \vee (p = \perp \wedge J_1 \wedge J_2)) \Rightarrow$

$(\neg Cycle \wedge Successors = Successors^0) \vee ((Cycle \vee Successors = Successors^0) \wedge Successors = \emptyset)$

$\equiv$ {$((A \wedge B) \vee (C \wedge D) \vee E) \Rightarrow (A \vee C)$}

$true$

What it is left to prove is that invariants J_1 and J_2 hold at the beginning of the while loop, and that they are preserved by the loop.

At the beginning of the loop (between line 6 and line 7) we have:

J_1

$\equiv$ { definition of J_1}

$\neg progress \vee (\neg Cycle \wedge Successors = Successors^0)$

$\equiv$ {$\neg progress$ because $progress$ was set to $false$ in line 7}

$true$

We also have:

J_2

$\equiv$ {definition of J_2}

$Successors \neq \emptyset \vee Cycle \vee Successors = Successors^0$

$\equiv$ {$Successors = Successors^0$ because $Successors$ was never changed}

$true$

So both J_1 and J_2 hold at the beginning of the while loop, and we are left to prove that they are maintained by the loop body.

First, we notice that lines 8, 15, 16, and 17 from the loop body cannot change the values of J_1 and J_2, if they hold before executing those lines they have to hold after executing those lines. We only have to look at lines 10 and 13.

Let's assume that right before line 10 P_1 holds, with $P_1 \equiv \neg Cycle \wedge Successors = Successors^0$. We know that $progress = true$ cannot change the value of P_1, so P_1 is also true after executing line 10. We know that $P_1 \Rightarrow J_1 \wedge J_2$, so line 10 preserves J_1 and J_2.

Let's assume that right before line 13 P_2 holds, with $P_2 = \neg progress \wedge Cycle$. Executing line 13, does not change P_2. We know that $P_2 \Rightarrow J_1 \wedge J_2$, so line 13 preserves J_1 and J_2.

We are left to prove that P_1 holds right before line 10 and that P_2 holds right before line 13.

Let's define $J_3 \equiv Successors = Successors^0 \vee Successors = \emptyset$. J_3 holds before the while loop because the variable $Successors$ is not changed. J_3 is maintained by the loop body because the only line that changes $Successors$, changes it to $\emptyset$, keeping J_3 true. So J_3 is also a loop invariant.

Right before line 10 we know that the loop guard holds and also J_3 holds. We prove that $G \wedge J_3 \Rightarrow Successors = Successors^0$.

$G \wedge J_3 \Rightarrow Successors = Successors^0$

$\equiv$ {definitions of G and J_3}

$(\neg progress \wedge p \neq \perp \wedge Successors \neq \emptyset \wedge (Successors = Successors^0 \vee Successors = \emptyset)) \Rightarrow Successors = Successors^0$

$\equiv$ {distributivity of $\wedge$ over $\vee$; $P \wedge \neg P \equiv false$}

$(\neg progress \wedge p \neq \perp \wedge Successors \neq \emptyset \wedge Successors = Successors^0) \Rightarrow Successors = Successors^0$

$\equiv$ {$P \wedge Q \Rightarrow P$}

$true$

Right before line 13 we know that the loop guard holds, and so we have $\neg progress$.

We are left to prove that right before line 10 we have $\neg Cycle$, and that right before line 13 we have $Cycle$.

n was obtained by extending u_0 with the sequence $q = a_1 a_2 ... a_k$. Let $q[i : j] = a_i ... a_j$ if $0 < i \leq j \leq k$, and $q[i : j] = \epsilon$ otherwise. Let a_i be the current action a in the loop body. Initially $a_i = a_k$.

We define a new invariant J_4 stating that $q[i + 1 : k]$ does not exhibit a local execution cycle (Definition 24).

J_4 trivially holds at the beginning of the while loop since $q[k + 1 : k] = \epsilon$.

To prove that J_4 is maintained by the loop body, we only have to look at line 16, since it is the only line that changes the current action a.

Right before line 16 we have (from the conditions in the if statements):

$\neg MadeProgress(a, t) \wedge \neg conflicts(a, A, t) \wedge (last\_thread(p) \neq t \vee LocalState(p) \neq LocalState(n))$

Assuming that $q[i+1 : k]$ does not exhibit a local execution cycle, we prove that $q[i : k]$ does not exhibit a local execution cycle either.

To exhibit a local execution cycle, p would have to correspond to u_i in Definition 24, otherwise $q[i + 1 : k]$ would have exhibited a local execution cycle.

If $last\_thread(p) \neq last\_thread(n)$, then p cannot correspond to u_i in Definition 24 and $q[i:k]$ does not exhibit a local execution cycle.

If $last\_thread(p) = last\_thread(n) \wedge LocalState(p) \neq LocalState(n)$, then we try to repeatedly swap, according to the dependency relation, actions from other threads so they end up before a_i. If the swapping fails for at least one action, we are done since we cannot form a local execution cycle.

Let's assume that the swapping succeeds for all the actions from the other threads. We are left with a tail that starts at a state representation u with a_i the next action and contains only actions from $last\_thread(n)$.

Because all the actions between p and u are from other threads than $last\_thread(n)$ we know that the local state for this thread has not changed between p and u. We have $LocalState(p) \neq LocalState(n)$, so there cannot be a local execution cycle in A_G between states corresponding to u and n. If there were such a local execution cycle then we would have had $LocalState(p) = LocalState(n)$.

We have established that J_4 is maintained by the loop body.

We need another invariant: $J_5 \equiv \forall a_j \in A.\neg MadeProgress(a,t) \wedge \neg conflicts(a_j, A, t)$.

J_5 trivially holds at the beginning of the loop since $A = \emptyset$.

A is changed only in line 15, by adding a new action a. From the guards of the if statements, right before line 15, we have:
$\neg MadeProgress(a,t) \wedge \neg conflicts(a, A, t)$ holds. From Definition 23 we get that $\neg conflicts(a, A \cup \{a\}, t)$, so the invariant J_5 is maintained by the loop body.

We need another invariant: $J_6 \equiv \forall j.i < j \leq k.a_j \in A$.

J_6 trivially holds at the beginning of the loop since $i = k$.

A is changed only in line 15, by adding the new action $a = a_i$. We get that $\forall j.i-1 < j \leq k.a_j \in A$.

Right after line 16, i has decreased by 1 (we have moved up one action), but J_6 still holds since we have added a to A one line above.

We are now ready to prove that right before line 10 $\neg Cycle$ holds.

Right before line 10, from the guard of the if statement we have:
$MadeProgress(a,t) \vee conflicts(a, A, t)$.

If $MadeProgress(a,t)$ is $true$ then action a is from thread t and has changed a global variable. No local execution cycle can include action a and so now we can declare that $\neg Cycle$ holds.

If $conflicts(a, A, t)$ is $true$ and $thread(a) = t$, then there exists $a' \in A$, $thread(a') \neq t$, such that $(a, a') \in D$. From invariant J_4 we know that if there exists a local execution cycle, then a has to be part of it. To exhibit this local execution cycle, we should be able to swap a with any action that follows it and is from another thread. From invariant J_6 we know that A contains all the actions that follow a, so a' follows a, but we cannot swap them because $(a, a') \in D$. So in this case $\neg Cycle$ holds.

If $conflicts(a, A, t)$ is $true$ and $thread(a) \neq t$, then there exists $a' \in A, thread(a') = t$ such that $(a, a') \in D$. From invariant J_6 we know that A contains all the actions that follow a, so a' follows a. Any local execution cycle that ends in the state represented by n would have to include a (from invariant J_4). The Definition 24 requires that $(a, a') \notin D$ which in not possible in this case.

We can now prove that right before line 13 $Cycle$ holds.

Right before line 13, from the guard of the if statements we have:
$\neg MadeProgress(a, t) \wedge \neg conflicts(a, A, t) \wedge last\_thread(p) = t \wedge LocalState(p) = LocalState(n)$.

From $last\_thread(p) = last\_thread(n)$ and invariants J_5 and J_6 we get that we can reorder the actions from p to n in such a way that Definition 24 is satisfied except for the part with the local execution cycle in A_G.

We know there exists an u_j between p and n such that all the actions between p and u_j are from other threads than t, and also all the actions between u_j and n are from t. Also, we know that these groups of actions are independent with respect to each other.

From p to u_j the local state of thread t is not changed since no action is from t. From u_j to n we have actions only from thread t so the local state of the other threads cannot be changed. We also have that between u_j and n no communication variable was modified. It follows from Definition 3 that the state $s \in A_G$ that corresponds to u_j is the same as the state $s' \in A_G$ that corresponds to n. All the actions between s and s' are from thread t and none of them change communication variables, so according to Definition 20 we have found a local execution cycle.

Now, using Definition 24, we get that the sequence of actions that led to n exhibits a local execution cycle. □

Theorem 8. *If all the execution paths in A_G are either finite(no cycles), or partial order equivalent to some other executions that involve only local execution cycles, then Algorithm 2, with the list $Filters$ containing only the functions presented in Algorithms 3 and 5 (in this order), terminates and explores all the states in A_G.*

Proof. Algorithm 5 does not allow the exploration of infinite paths. Any infinite path has a finite length prefix that exhibits a local execution cycle. According to Lemma 13, such a local execution cycle would be identified by Algorithm 5 and $Successors$ would be set to $\emptyset$ effectively stopping the exploration of that path at that point. Since only finite paths are explored, and A_R contains no cycles, Algorithm 2 must terminate.

For any state $s \in A_G$ with $s_0 \stackrel{w}{\Rightarrow} s$, let w' be a sequence of transitions that is partial order equivalent to w and in which all the unrolls of a local execution cycle are grouped together and don't have any transitions from other threads (Definition 20). By removing from w' each complete unrolling of a local execution cycle we are left with a sequence w''. We have $s_0 \stackrel{w''}{\Rightarrow} s$ and w'' does not contain any local execution cycles. By taking $q = abstract(w)$, the sequence of actions that

leads from s_0 to s in A_G, we have that q does not exhibit a local cycle, and so it will not be cut by Algorithm 5. If q is not cut by Algorithm 3, then a state n, that corresponds to u_0 being extended with q, is explored. If q is cut by Algorithm 3, then we can construct q' partial order equivalent to q such that q' also leads to s (see Theorem 6). □

4.2.4 The *SuperStepPOR* Filter

The *SuperStepPOR* filter implements the partial order reduction introduced in [27, 26].

Definition 25. A sub-graph $A_S = (V_S, E_S)$ of a state representation graph $A_R = (V_R, E_R)$ is called a *super step reduced graph* of A_R if it has the following properties:

- $V_S \subseteq V_R$ is the set of nodes from A_S.
- $V_S = V_{SN} \cup V_{IN}$, where V_{SN} is the set of super nodes from A_S, and V_{IN} is the set of intermediate nodes from A_S. V_{SN} and V_{IN} are not necessarily disjoint.
- $u_0 \in V_{SN}$, the representation of the initial state s_0, is a *super node*.
- a *super step* ss is a path $u_1 \xrightarrow{a_1} u_2 \xrightarrow{a_2} ... \xrightarrow{a_{n-1}} u_n$ from A_S such that $u_1 \in V_{SN}$, $u_n \in V_{SN}$, for all u_i, $1 < i < n$, $u_i \in V_{IN}$, and there exists a thread t such that for all a_i, $0 < i < n$, $thread(a_i) = t$.
- for a super step $ss = u_1 \xrightarrow{a_1} u_2 \xrightarrow{a_2} ... \xrightarrow{a_{n-1}} u_n$, we call $\{a_i \mid 0 < i < n-1\}$ the *set of intermediate actions* of the super step ss.
- for a super step $ss = u_1 \xrightarrow{a_1} u_2 \xrightarrow{a_2} ... \xrightarrow{a_{n-1}} u_n$, we call a_{n-1} the *last action* of the super step ss.
- any super node $u \in V_{SN}$ has an associated set $SuperSteps(u)$ that contains the outgoing super steps from u.
- for any super node $u \in V_{SN}$, that corresponds to a state s in A_G, and for each action $a \in enabled(s)$, there exists a super step $ss(thread(a)) \in SuperSteps(u)$ that is taken by $thread(a)$.
- for any super node u, and for any super step $ss \in SuperSteps(u)$, all the actions of ss are independent with all the *intermediate actions* of all the other super steps in $SuperSteps(u)$.
- for any super step ss, none of its intermediate actions enables a previously disabled action from another thread.

Lemma 14. *Let A_R be the graph of representations corresponding to A_G, and let A_S be a super step reduced graph of A_R. Assuming that A_G is acyclic, for any state representation u from A_R reachable from u_0 by a sequence of actions $q = a_1 a_2 ... a_n$, there exists a sequence of actions $q' = a'_1 a'_2 ... a'_n$ that*

is partial order equivalent to q, and there exists an i, $0 \leq i \leq n$, such that the following properties hold:

- $u_0 \xrightarrow{a'_1} u_1 \xrightarrow{a'_2} ... \xrightarrow{a'_i} u_i$ *is a path from* A_S *and* u_i *is a super node in* A_S*;*
- *for any thread t, if* $b^t_1 b^t_2 ... b^t_k$ *is the sequence of those actions taken by t from the sequence* $a'_{i+1} ... a'_n$*, then for all* l, $0 < l \leq k$, *the path* $u_i \xrightarrow{b^t_1} ... \xrightarrow{b^t_l} u^t_{i+l}$ *does not end in a super node from* A_S *(*$u^t_{i+l} \notin V_{SN}$*).*

Proof. We prove the lemma by induction on the depth of u, which is the same as the length of q.

For $n = 1$, $q = \epsilon$ we have $q' = \epsilon$ and $i = 0$. The two properties are trivially satisfied since $u_0 \in V_{SN}$.

For $n = 1$, $q = a_1$ we have $q' = a_1$. Because u_0 is a super step end, and a_1 is enabled in u_0, there exists a super step starting from u_0 that contains a_1. We have two cases:
a) u_1 is a super node, and in this case $i = 1$. The two properties hold since $u_0 \xrightarrow{a_1} a_1$ is a path from A_S ending in a super node, and the suffix $a_i ... a_n$ is empty.
b) u_1 is not a super node, and in this case $i = 0$. The first property trivially holds since u_0 is a super node, and the second property holds because u_1 is not a super node.

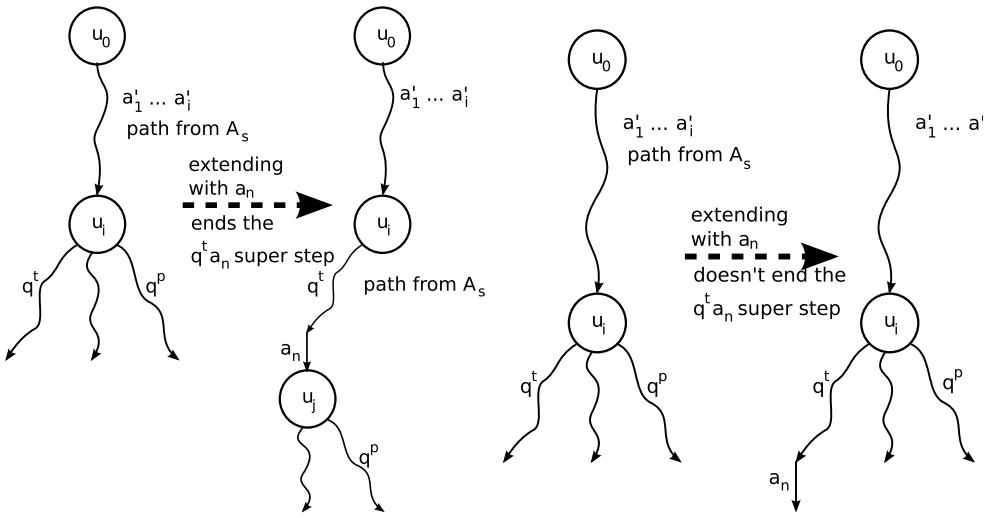
We have proved the base case, and now we proceed to the induction step.

Let $q = a_1 ... a_{n-1} a_n$. From the induction step we know there exists a path $a'_1 ... a'_{n-1}$ and an i such that the two properties hold for $a_1 ... a_{n-1}$.

Let q^t be the sequence of actions taken by $thread(a_n)$ starting from u_i. From the induction hypothesis we know that q^t doesn't pass through a super node in A_S. Again, we have two cases:
a) The sequence $q^t a_n$ leads to a super node in A_S (the case depicted in Figure 4.2a). From the induction hypothesis we know that for any thread $p \neq t$, q^p doesn't pass through a super node, and so we must have that $q^t a_n$ is independent with all the actions in q^p. If this were not true, then considering Definition 25, either q^t or q^p would have to pass through a super node, thus violating the induction hypothesis.

Because for any thread $p \neq t$ we have that $q^t a_n$ is independent with q^p, we can reorder q to get a partial order equivalent sequence q' with $q' = a'_1 ... a'_i q^t a_n ... a''_n$ such that $a^t a_n$ starts from u_i and ends in a super node u_j. We now have that $a'_1 ... a'_i q^t a_n$ is a path from A_S. We also know from the induction hypothesis that for any two threads t_1 and t_2, q^{t_1} is independent with q_{t_2} (otherwise one of them would have to pass through a super node starting from u_i). If started from u_j, any q^p passes through a super node, we can reorder the suffix such that the part of q^p that forms the super step appears right after u_j. We repeat this process until the two properties hold.

b) The sequence $q^t a_n$ does not lead to a super node in A_S (the case depicted in Figure 4.2b). $q' = a'_1 ... a'_{i-1} a_n$ and i satisfy the two properties. Using the same argument as in case a) we have



(a) a new super node is reached (b) no super node is reached

Figure 4.2: Extending a path with an action a_n

that for any two threads t_1 and t_2, q^{t_1} and q^{t_2} are independent starting from u_i. a_n has to be independent with all the actions from other threads, since otherwise it would imply that either $q^t a_n$ or another q^p passes through a super node. □

Theorem 9. *Let A_R be the graph of representations corresponding to A_G, and let A_S be a super step reduced graph of A_R. Assuming that A_G is acyclic, if a state s in A_G, having a representation u in A_R, violates a safety property then there exists a state representation u' in A_S that corresponds to a state s' in A_G such that s' violates the same safety property.*

Proof. By applying Lemma 14 we can construct a sequence of actions $q = a_1...a_i...a_n$ that starting from u_0 leads to u such that $a_1...a_i$ is a path in A_S starting from u_0 and none of the actions a_j, $i < j \leq n$ is the end of a super step.

If u corresponds to a deadlock state then all the threads are disabled in u. Any action from a thread t, that leads to a state in which t is disabled has to be the last action of a super step (we can't continue the super step). So u is reached by the last action of a super step, so u has to be a super node, and in this case $u' = u$.

If u corresponds to an assertion violation or a global invariant violation then none of the actions a_j, $i < j \leq n$ can change the value of the assertion or the value of the global invariant because otherwise a_j would conflict with all the other actions and so it would have to be the end of a super step. So the same assertion violation or global invariant violation is present in u_i reached by taking the sequence of actions $a_1..a_i$ from u_0. So in this case we have $u = u'_i$. □

The *SuperStepPOR* filter tags to each state representation u the super step $SuperStep(u)$. This means that $last\_action(u)$ is part of $SuperStep(u)$ as either an intermediate action or as the last

Algorithm 6 *SuperStepPOR*(*Successors*)

```
1:  ss_end = true
2:  NextRep = {n' ∈ Successors | last_thread(n') = last_thread(n)}
3:  if n ≠ u_0 ∧ NextRep ≠ ∅ then
4:     ss = SuperStep(n)
5:     sn = SuperNode(ss)
6:     next = choose(NextRep)
7:     if IndepLast(ss, sn) ∧ IndepInter(next, sn)∧
          ¬Enables(last_action(n), Successors \ NextRep) then
8:        ss_end = false
9:        Intermediate(ss) = Intermediate(ss) ∪ {Last(ss)}
10:       Last(ss) = last_action(next)
11:       SuperStep(next) = ss
12:       Successors = NextRep
13:    end if
14: end if
15: if ss_end then
16:    sn = new SuperNode
17:    SuperSteps(sn) = ∅
18:    for all n' ∈ Successors do
19:       ss = new SuperStep
20:       SuperNode(ss) = sn
21:       Intermediate(ss) = ∅
22:       Last(ss) = last_action(n')
23:       SuperSteps(sn) = SuperSteps(sn) ∪ {ss}
24:       SuperStep(n') = ss
25:    end for
26: end if
27: return Successors
```

action.

With each super step ss we associate its set of intermediate actions $Intermediate(ss)$, its last action $Last(ss)$, and the parent super node $SuperNode(ss)$.

A super node is represented by the set of outgoing super nodes, and when it is created, it marks the fact that the current state representation n is in V_{SN}.

The main idea of the *SuperStepPOR* filter is to create new super steps for each outgoing action from a super node u. Initially these new super steps contain only the corresponding action that was enabled in u. Each super step is repeatedly extended as long as it can be extended. When the super step cannot be extended any longer, a new super node is created at the end of the super step, and the same process is applied to the new super node.

$IndepLast(ss, sn)$, in line 7 of the filter, tests if the last action from the super step ss is independent with all the last actions from other sibling super steps taken from the super node sn.

$IndepInter(next, sn)$, in line 7 of the filter, tests if $last\_action(next)$ is independent with all the intermediate actions from super steps taken from the super node sn, except the super step that corresponds to $last\_thread(next)$.

$Enables(last\_action(n), Successors \setminus NextRep)$ determines if the last taken action enabled actions from threads that were not previously enabled.

Theorem 10. *Algorithm 2 with Filters holding only Algorithm 6 terminates and traverses A_S, a super step reduced graph of A_R.*

Proof. The filter does not contain any loops so it has to terminate. The variable $Successors$ is modified only on line 12 where it becomes a subset of the initial value for $Successors$. Algorithm 2 is a restricted graph exploration, and because A_G is acyclic, it calls the $SuperStepPOR$ filter only a finite number of times. So Algorithm 2 terminates.

For u_0, Algorithm 6 creates a new super node. The guard of the if statement in line 3 is false when $n = u_0$ and this ensures that the variable ss_end is $true$ in line 15. So for $n = u_0$, lines 16 to 25 of the algorithm are executed. These lines create a new super node that corresponds to u_0 and also create new super steps for each enabled action in u_0.

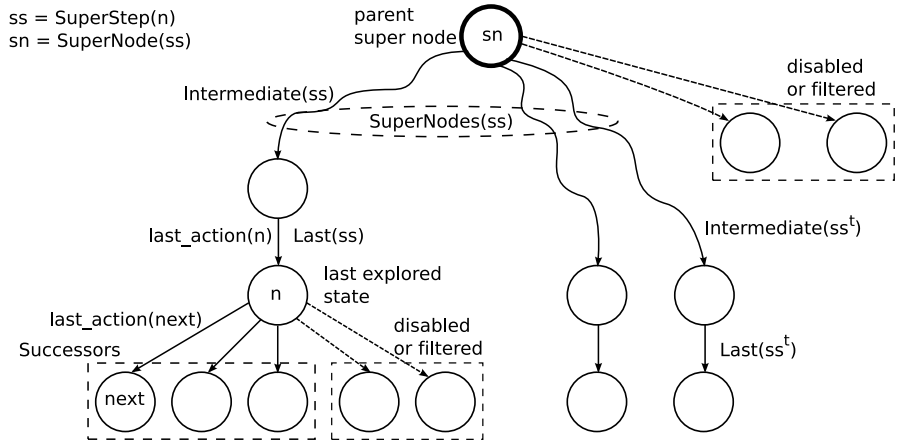


Figure 4.3: The super step partial order reduction

Figure 4.3 presents the data structures used by the $SuperStepPOR$ filter, and what are their connections to each other.

The current state representation n is part of a super step $ss = SuperStep(n)$ taken by a thread $i = last\_thread(n)$ from the parent super node $sn = SuperNode(ss)$.

The algorithm maintains the following invariant J_1: for any thread $t \neq last\_thread(n)$, $Intermediate(ss)$ is independent with $Intermediate(ss^t)$, and also $Last(ss)$ is independent with $Intermediate(ss^t)$, where ss^t is the super node taken from sn by thread t.

The algorithm also maintains the invariant J_2 none of the actions in $Intermediate(ss)$ enable actions from other threads that were previously disabled.

Together, these two invariants ensure that the conditions presented in the super step definition (Definition 25) are not violated during the exploration.

The $SuperStepPOR$ filter tries to extend $ss = SuperStep(n)$ by adding the next action from $last\_thread(n)$ to the super step. First, line 3, ensures that n is not u_0 so there exists a last action. Second, line 3, also ensures that there exists a next action from the same thread ($NextRep \neq \emptyset$, there exists a successors state representation reached by taking an action from the same thread).

$IndepLast(ss, sn)$ and $IndepInter(next, sn)$, if true, ensure that the invariant J_1 is maintained by extending the super step with the action $last\_action(next)$.

$\neg Enables(last\_action(n), Successors \setminus NextRep)$, if true, ensures that the invariant J_2 is maintained by extending the super step with the action $last\_action(next)$.

If the super step can be extended, then we can remove from $Successors$ (line 12) all the successor state representations that are not reached by taking an action from the current thread.

If it is determined that the super step cannot be extended ($ss\_end = true$), then to maintain the invariants J_1 and J_2, n has to be the end of the super step ss and become a super node. This is exactly what happens in lines 16-25: a new super node corresponding to n is created, and also new outgoing super steps are created for all the actions that are enabled in n (for all the successors state representations).

The conditions presented in Definition 25 are not violated during the algorithm execution meaning that the algorithm explores A_S, a super step reduced graph of A_R. □

Theorem 11. *If a state s in A_G violates a safety property, then Algorithm 2 with Filters holding only Algorithm 6 terminates and explores a state s' from A_G, with s' violating the same safety property as s.*

Proof. We apply Theorem 10 and we get that Algorithm 2 terminates and explores A_S, a super step reduced graph of A_R.

We apply Theorem 9 and we get that any safety violation that would be exposed by exploring A_R is also exposed by just exploring A_S. □

Chapter 5

SCALE - Design and Implementation

SCALE is divided into three major components. First, a *Static Analysis And Code Instrumentation* component finds, by analyzing the source code, the global statements and modifies the code around them to allow an external scheduler to control the way threads are interleaved. Second, a *Scheduling* component, explores the application's state space by trying many different thread schedulings. During exploration, when stepped, a thread atomically executes the code between two consecutive global statements. SCALE checks if every visited state satisfies the specified properties. If a violation is found, then a trace of how to get to the error state is generated. Finally, a third *Simulation* component replays error traces allowing the bugs to be reproduced and analyzed at a later time.

SCALE's static analysis and code instrumentation is written in OCaml, has about 300 lines of code, and was developed as a CIL [20] code transformation. The other components were developed in C and have around 11000 lines of code, most of which is for the scheduling component. SCALE was tested on x86, 32 bit and 64 bit, Linux platforms, and it should be easily ported to any platform that has: a C compiler, an OCaml compiler, the pthread library, and a mechanism for turning off Address Space Layout Randomization[8, 1, 3].

5.1 Static Analysis and Code Instrumentation

The purpose of SCALE's static analysis and code instrumentation is to modify the verified program such that an external *Scheduler* process can control how the program's threads are interleaved. Using CIL's[20] points-to analysis and then a CIL transformation, we identify and replace the global statements described in section 3 with wrapped versions. Besides having the same functionality the original ones, the wrapped versions of the global statements, also implement the instrumented executable's side of the *Scheduling Protocol* that will be presented in Section 5.2. If finer control is needed, the wrappers can also be inserted manually. There are wrappers for each kind of global statement: creating a new thread, locking and unlocking a mutex, reading and writing to a variable, etc. The wrappers are grouped together in a wrapper library so they can be easily reused. The

whole process is presented in Figure 5.1. At the end, we get an *instrumented executable* that uses the *Scheduling Protocol* to communicate with a *Scheduler*.

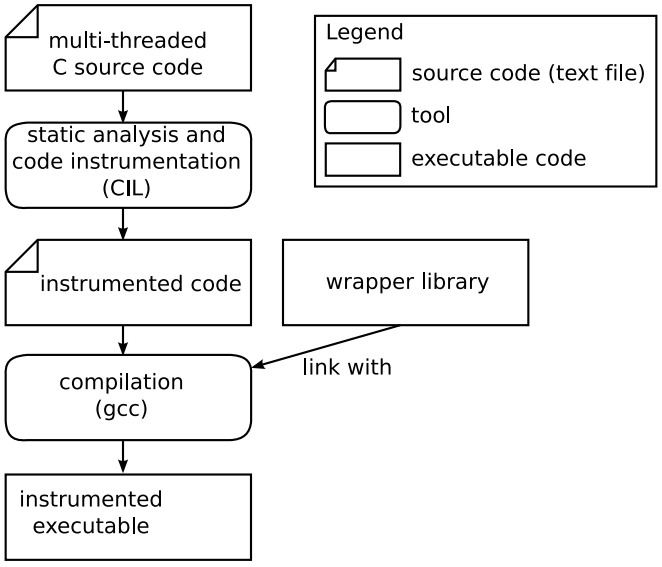


Figure 5.1: SCALE's static analysis and code instrumentation process

5.2 The SCALE Scheduling Protocol

The *Scheduling Protocol* describes the communication between the *instrumented executable* and the *Scheduler*. As a consequence of this communication, the *Scheduler* is able to control the way the *instrumented executable*'s threads are interleaved and also to inspect parts of the *instrumented executable*'s state.

5.2.1 Communication Channels

For each thread of the instrumented executable, there exists a set of two communication channels between the scheduler and that thread. One channel is used to send *commands* from the scheduler to the thread, and the other channel is used to send *command results* from the thread back to the scheduler. The interaction between the scheduler and the instrumented executable is depicted in Figure 5.2.

In each of the instrumented executable's threads, each wrapped global statement contains, right before the original statement, a loop that reads commands sent by the scheduler. During execution, the application can be seen as a collection of threads, each blocked right before a global statement, waiting for commands. A thread step happens when the scheduler unblocks one of the threads. The unblocked thread exits the current loop and executes up to the next wrapped global statement where

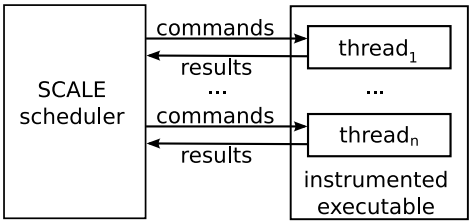


Figure 5.2: Scheduling protocol overview

it blocks again in a new loop. The process is repeated until the application ends or it is canceled by the scheduler.

5.2.2 Commands and Command Results

There are two types of commands: *query commands* and *control commands*. The scheduler uses *query commands* to get relevant information about the current state of the application, and *control commands* to either move the application to a successor state by scheduling a thread in the current state, or to exit the application. To allow multiple queries for the current state, commands are received in a loop. While query commands leave the thread who receives them in the same loop, control commands exit the current loop, and cause the execution to be either continued or aborted. Sending a command and receiving the result is done using synchronous communication and resembles a synchronous remote procedure call.

GetNextAction is a query command that asks a thread to report its next action (Definition 10) in the current state. The result structure that is sent back from the thread to the scheduler contains the following information:

- the identifier of the blocked thread;
- the operation performed by the global statement. It can be one of the following:
  - program start: the main thread is created and the execution starts in the initial state;
  - program exit: the program ends, and there are no more statements that can be executed;
  - thread creation: a new thread is created, the scheduler must know how to communicate with it;
  - thread start: a thread starts executing its statements, it is now active in the application;
  - thread exit: a thread is exited so its statements can no longer be executed;
  - thread join: wait for another thread to exit before proceeding;
  - read or write a global variable: access to a global variable;
  - synchronization primitive call: one for each primitive, for example locking or unlocking a mutex, waiting on a condition variable, etc.;

  - assertion: tests on the current state of the application that were specified in the source code;

- the list of communication variables that will being accessed by the global statement if the thread is scheduled in the current state. As described in Section 3, in SCALE, a communication variable is one of the following: a thread identifier, the memory location of a shared variable, a mutex handle, or a condition variable handle.

GetCurrentBackTrace is a query command that asks a thread to report its current function call stack. The result contains the following information:

- number of stack frames: how many functions are in the call stack, and also how many control points does the result contain;
- array of control points: one control point for each entry in the call stack. A control point is an abstraction of a stack frame and contains the following information:
  - level: the stack frame number, 0 for the top of the stack, 1 for the second one, and so on;
  - address: the address of the current instruction in the function's code;
- stack hash: a hash of the current contents of the stack.

EvaluateFunctionCall is a query command that asks a thread to call a function in its current context and send the result back to the scheduler. The command contains the name of the function to be evaluated. The function takes no arguments and returns an integer. The result of the function call is sent back to the scheduler as a command reply. The *Evaluate Function Call* command is used for checking whether invariants are satisfied or not in the current state.

Step is a control command that asks a thread to exit the current command loop and start executing statements until the next global statement. The returned result is the next action corresponding to the newly reached global statement. The command spans between two consecutive global statements since its result can only be sent after the new global statement is reached. SCALE solves this problem by requiring the thread, right before entering a command loop, to send the scheduler a reply containing its next action.

Exit is a control command used by the scheduler to abort the current execution of the application. The receiving thread sends an acknowledgment back to the scheduler and then forcefully causes the the whole application to exit.

5.3 The SCALE Scheduler

This section describes how SCALE's *Scheduler* implements the algorithms presented in Chapter 4. These algorithms guide the state space exploration and check if any properties are violated.

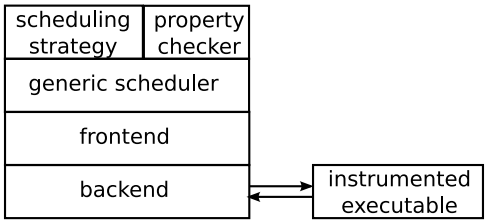


Figure 5.3: The SCALE Scheduler

As presented in figure 5.3, the scheduler has a layered implementation. At the lowest layer it has a component that communicates with the instrumented executable and allows the higher layers to schedule individual threads. The higher layers keep track of what states have been visited and decide what new states should be explored next. The following subsections describe these layers in more detail.

5.3.1 Backend

The *Backend* is a thin layer that completely abstracts away the communication with the instrumented executable's threads. Together with the *wrapper library*, it implements the scheduling protocol described in Section 5.2. The sending and the receiving of scheduling commands and results are wrapped and presented to the layers above as API functions.

Next is a description of the *Backend* API.

- $bk\_init()$: spawns a new process with the instrumented executable (which should block before the first global statement), and returns the next action for the main thread;
- $bk\_exit()$: causes the application to be exited;
- $bk\_restart()$: aborts the current run of the application, restarts its execution (which should block before the first global statement), and returns the next action for the main thread;
- $bk\_step\_thread(t)$: schedules thread t, runs it up to its next global statement, and returns the next action for thread t;
- $bk\_get\_back\_trace(t)$: returns the current back trace of thread t;
- $bk\_eval\_function(t, func)$: evaluates the function $func$ in thread t's current context and returns the function's result.

5.3.2 Frontend

The *Frontend* is a layer built on top of the backend which keeps track of the current configuration of the application (Definition 12).

A configuration contains the following information:

- the state of each thread: keeps track of each thread's next action, and next step number.
- the state of each communication variable: each such variable is given a unique identifier. For every variable, the frontend keeps track of how many times it was written so far. We call this number the current version of the variable, and it is used to differentiate accesses to the same variable.
- the state of each synchronization primitive: for each mutex it stores the current holder thread, and for each condition variable, it keeps track of the set of waiting threads.
- the thread that was scheduled in the last step: this is the current thread.
- the new thread that was created during the last step (if one was created): the new thread is now active, was added to the current configuration, and from now on it can be scheduled.

Given the current configuration, the set of enabled threads and their next actions can be easily computed.

The frontend exposes a higher level API than the backend.

- $fr\_init()$: starts the execution of the application and returns the initial configuration;
- $fr\_exit()$: aborts the current run of the application;
- $fr\_restart$: aborts the current run of the application, restarts the application, and returns the initial configuration;
- $fr\_step\_thread(t)$: steps thread t up to the next global statement, updates the current configuration accordingly, and returns the current configuration;
- $fr\_get\_back\_trace()$: returns the result of the same backend API function, called for the last scheduled thread;
- $fr\_check\_invariant(func)$: takes an invariant name as argument and uses the backend's $bk\_eval\_function$ API call to determine if the given invariant is satisfied or not in the current thread's context.

5.3.3 Generic Scheduler

Algorithm 7 presents how the *Generic Scheduler* glues together the other parts of the state space exploration to implement the algorithms described in Chapter 4.

Algorithm 7 The Generic Scheduler

```
1:  config = fr_init()
2:  while true do
3:    if violates_properties(config) then
4:      report_error()
5:      break
6:    end if
7:    dec = strategy_decide(config)
8:    if dec = Schedule(t) then
9:      config = fr_step_thread(t)
10:   else
11:     if dec = Restart then
12:       config = fr_restart()
13:     else
14:       break
15:     end if
16:   end if
17: end while
18: fr_exit()
```

All scheduling decisions are deferred to a *scheduling strategy* by calling the *strategy_ decide* function. The strategy can decide to schedule a thread from the current state, restart the verified application from the initial state, or end the exploration. All verifications are deferred to the *property checker* by calling the *violates_ properties* function.

At any moment, the generic scheduling algorithm stores only the current configuration and has no mechanism for back tracking to a previously visited state.

5.3.4 Scheduling Strategy

The *Scheduling Strategy* is the component responsible for representing and storing visited states as *happens-before graphs for actions* (Definition 11).

Similarly to [19], we represent a happens before graph as a set of actions that have been extended with information about the edges.

Each such extended action is a tuple: $< tid, step, op, al >$ where:

- *tid*: is the identifier of the thread taking the action;
- *step*: counts how many actions were taken by thread *tid* so far;
- *op*: the operation performed by the action;
- *al*: is a list of accesses to global variables. Each access is a tuple $< v, at, w >$, where v is the variable identifier, at is the type of access and can be either $READ$ or $WRITE$, and w is a counter that gets incremented with each $WRITE$. A $READ$ refers to the last w value.

There is an edge from an extended action $< tid_1, step_1, op_1, al_1 >$ to an extended action $< tid_2, step_2, op_2, al_2 >$ if any of the following is true:

- $(tid_1 = tid_2) \wedge (step_1 < step_2)$;
- $\exists (v_1, at_1, w_1) \in al_1$ and $\exists (w_2, at_2, w_2) \in al_2$ such that $(v_1 = v_2) \wedge (at_1 = WRITE) \wedge (w_1 \leq w_2)$;
- $\exists (v_1, at_1, w_1) \in al_1$ and $\exists (w_2, at_2, w_2) \in al_2$ such that $v_1 = v_2 \wedge (at_1 = READ) \wedge (w_1 < w_2)$.

An advantage of this state representation is that the state space is stored efficiently, by constructing states incrementally, a child state is stored as the last action plus a link to the parent state. Moreover, as presented in[19] we can compute a hash of a state incrementally in the same way. The hash of a child state is the hash of the parent state xor'ed with the hash of the last action. Since xor is commutative, the same set of actions will have the same hash no matter in which order they are xor'ed. By comparing the hashes, we get a fast, but unsound, comparison between states. Also, storing the states incrementally makes some reduction algorithms easier to write.

When the exploration algorithm requires to *replay* a state s, there are two situations:

- s is a successor of the current state reachable by executing an action a, and in this case the *strategy_decide* function returns $Schedule(t)$, where $t = thread(a)$;
- s is not a successor of the current state, and in this case the *strategy_decide* function returns *Restart*. Assuming that $a_1 ... a_n$ is a sequence of actions that leads from s_0 to s, subsequent calls of *strategy_decide* return in order $Schedule(t_i)$, where $t_i = thread(a_i)$. When s is reached the scheduling strategy resumes normal operation.

When the state space exploration is done, the *strategy_decide* function returns *Exit*.

5.3.5 Property Checker

The property checking is done by delegating the verification to specialized checkers, each looking for one of the following classes of errors: deadlocks, assertion failures, and invariant violations. The checkers are provided with the current configuration of the application, and are also allowed to evaluate expressions in the current application context. By default, SCALE searches for all of the above errors, except invariant violations. To also look for invariant violations, the user has to provide a property file that contains the names of the functions implementing the invariant tests. The functions should be callable from each thread, at any point and should return a boolean value, true if the invariant holds, false otherwise. SCALE reports all errors that it finds to the user, and also generates a trace file that contains all the scheduling decisions that were made, starting from the initial state, up to the error state.

5.4 Error Replay

The *Simulation* component runs the application and guides it on a path that is described in an input trace file. This component is used to replay errors discovered during verification.

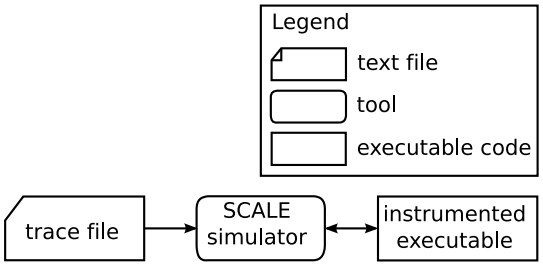


Figure 5.4: The SCALE Simulator

The simulator is a simple loop that reads lines, one by one, from the trace file and steps the corresponding threads. The trace file is a list of thread identifiers, separated by white spaces. When replaying the trace, the threads are scheduled in the order in which they appear in the file. The simulator provides detailed information about what happened in each step: the line of code where the step started, what operation was performed, and what variables were accessed.

Before stepping a thread some checks are done. First, the thread must be active in the application, and second, it must be enabled in the current state. The simulator reports an error if these two conditions are not met.

Chapter 6

Experiments and Preliminary Results

6.1 Non-Blocking Queue Algorithm

We have implemented a C version of the non-blocking queuing algorithm presented in[23] and we have managed to discover a previously unknown bug in it. The full source code of our implementation is listed in Appendix A.1. We have limited the system to a producer and a consumer communicating via a shared non-blocking queue of size N. The producer enqueues the integers from 0 up to $N-1$, and the consumer dequeues N integers.

The queue is represented as an circular array of elements. Two indexes are stored, $FRONT$, with $FRONT\ mod\ N$ being the position from where an item should be dequeued, and $REAR$, with $REAR\ mod\ N$ being the position where an item should be enqueued. The queue is empty when $FRONT = REAR$, and full when $REAR = FRONT + N$. With each enqueue $REAR$ should increase by 1, and with each dequeue, $FRONT$ should increase by 1. Each queue element, if it is in use then it stores a pointer to the item, and if it is not in use then stores a $NULL$ pointer.

Since it is a queue implementation, we expect the consumer to dequeue the same numbers that were enqueued by the producer and in the same order. We have found that this is not always true. Using SCALE to check the correctness of the algorithm's implementation, we have determined scenarios in which a queue element is lost, it is enqueued by the producer, but it is skipped and never dequeued by the consumer. For example, with a queue of size $N = 2$, there is an execution in which the consumer first dequeues 1 instead of 0. The correct behavior of the consumer would be to dequeue 0, then 1, and then terminate.

For any $N \geq 2$, SCALE finds the following counter example, or a similar one: The consumer starts a dequeue operation on an empty queue ($REAR = FRONT$), reads and stores the queue element x at $Q[FRONT\ mod\ N]$. Since the queue is empty, the element at that position contains a $NULL$ pointer. The consumer is preempted right before checking if $REAR = FRONT$. Next the consumer enqueues two items I_1 and I_2. The $REAR$ is updated and now we have $REAR = FRONT+2$. The consumer is scheduled, and checks if the queue is empty ($REAR \stackrel{?}{=} FRONT$) and notices that the

queue is not empty. It then compares the item pointer in its stale version of the queue element at $REAR \ mod \ N$ (x) and finds that this pointer is $NULL$. If this is the case, the consumer considers that there must be another consumer dequeuing that the item at that position at the same time. $REAR$ is incremented in an attempt to help this other (nonexistent) consumer. So the consumer skips the item I_1 and dequeues item I_2 instead.

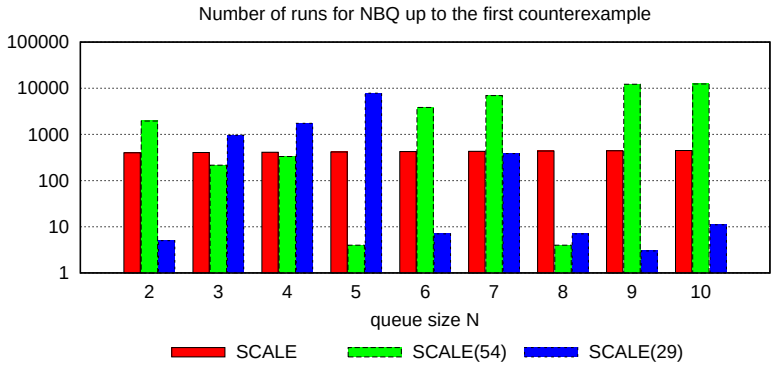


Figure 6.1: Number of runs up to the first counterexample for the *non-blocking queue* implementation

In figure 6.1 we plot how many runs were necessary to find a counter example for various queue sizes N. To show that the exploration order affects how fast the counterexample is found, we have used SCALE with 3 exploration algorithms:

(a) SCALE: the default DFS exploration which picks next actions in increasing order of the corresponding thread identifiers.

(b) SCALE(54): a DFS exploration that randomly picks the order in which the next actions are explored. The pseudo random number generator was seeded with the value 54.

(c) SCALE(29): same as (b), but with a seed of 29.

Note that not all runs are complete. SCALE restarts the program when it decides that running it further would not reach any unexplored states.

We have disabled the local cycle detection filter, and we have used only depth bounding, and we have measured the time required to restore a state as a function of the state's depth. Figure 6.2 presents the results that were obtained on an Ubuntu 9.04 Linux system, running as a virtual machine in Mac OS X, on an Intel Core 2 Duo 2.4GHz processor. As expected, the time required to restore a state increases with the state's depth. For large state depth values, the restore time becomes too large and the exploration is unfeasible.

We have configured SCALE to ignore the assertion violations in the non-blocking queue implementation, and we have measured the number of distinct state representations explored using different combinations of state filters. The results are presented in Figure 6.3. The non-blocking queue implementation contains busy-waiting loops, so if the local cycle detection filter is not used,

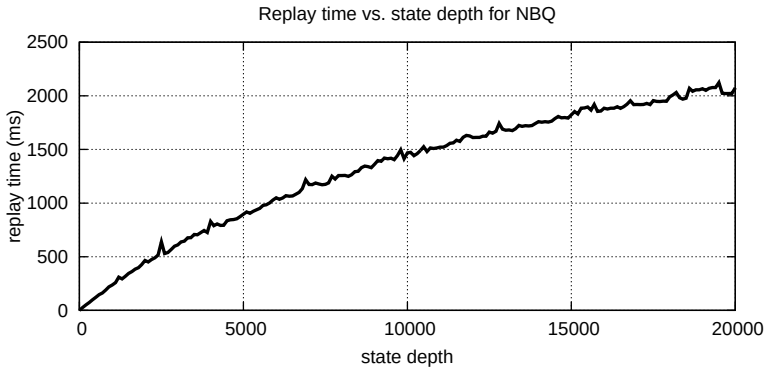


Figure 6.2: Replay time vs. state depth for the *non-blocking queue* implementation

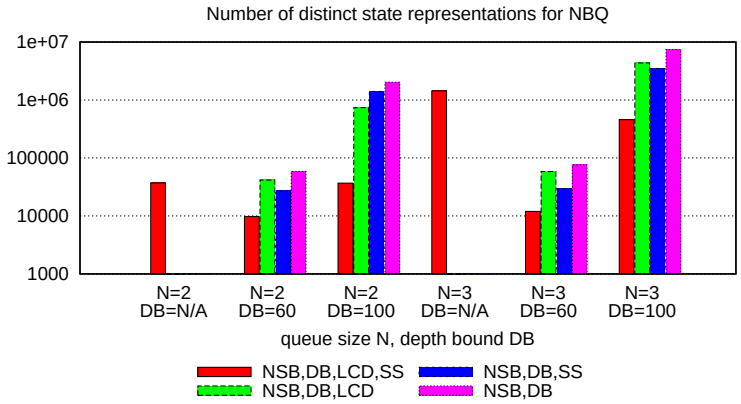


Figure 6.3: Number of distinct state representations for the *non-blocking queue* implementation

then the depth bounding filter must be used to ensure that the verification terminates. In Figure 6.3, NSB stands for the not seen before filter, LCD for the local cycle detection filter, SS for the super step partial order reduction filter, and DB for the depth bounding filter.

Figure 6.3 shows that using the super step partial order reduction filter in combination with the local cycle detection filter achieves a significant reduction compared to just using the not seen before filter and depth bounding.

Because he non-blocking queue implementation contains infinite executions we were unable to use Inspect to verify it. Inspect only works on terminating programs, and although it could bound the depth of the search, it doesn't have this feature implemented.

CHESS bounds the depth of the search, but when the depth bound is reached it reports a livelock. The execution that exhibits a livelock is unfair and would not be explored by a fair scheduler.

6.2 Indexer

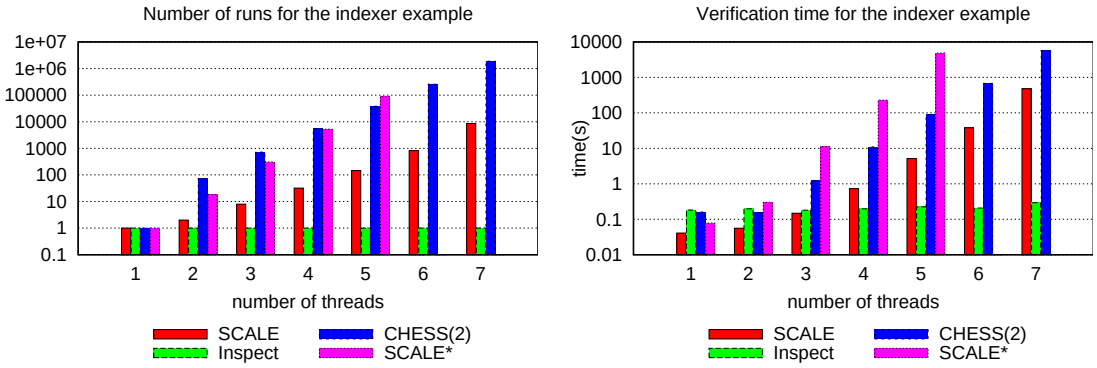
We have implemented the *indexer* example presented in [7] and compared SCALE's results to those of Inspect and CHESS. SCALE and Inspect use the same Linux version of the source code, while CHESS uses the Windows version. The source code for the example(both the Linux and Windows versions) is presented in Appendix A.2. The *indexer* example was introduced to show the benefits of using Dynamic Partial Order Reduction versus other partial order reductions where the independence relation between transitions is precomputed by a static analysis of the source code. In the example, each thread reads and writes an element of an array. The index of the element is computed dynamically, and depends on the thread identifier. A static analysis would fail to determine which threads would access the same element. The example is so constructed that no conflicts occur up to 11 threads.

CHESS, Inspect and SCALE compute the independence relation between transitions dynamically while running the program. If necessary, all three tools restart the program from the initial state to explore additional paths.

Tool	Threads	Runs	Time(s)	Tool	Threads	Runs	Time(s)
SCALE	1	1	0.041	SCALE	5	146	5.148
Inspect	1	1	0.182	Inspect	5	1	0.224
CHESS(2)	1	1	0.160	CHESS(2)	5	37927	88.110
SCALE*	1	1	0.078	SCALE*	5	89363	4863.917
SCALE	2	2	0.056	SCALE	6	818	37.881
Inspect	2	1	0.197	Inspect	6	1	0.207
CHESS(2)	2	73	0.156	CHESS(2)	6	257265	677.390
SCALE*	2	18	0.301	SCALE*	6	X	X
SCALE	3	8	0.147	SCALE	7	8593	478.146
Inspect	3	1	0.180	Inspect	7	1	0.294
CHESS(2)	3	715	1.234	CHESS(2)	7	1885380	5663.672
SCALE*	3	309	11.229	SCALE*	7	X	X
SCALE	4	32	0.728				
Inspect	4	1	0.197				
CHESS(2)	4	5448	10.937				
SCALE*	4	5256	228.827				

Table 6.1: Verification results for the *indexer* example

Table 6.1 presents the results obtained for different number of threads. We have measured the number of runs performed by each tool and the time necessary to complete the search. CHESS was run with a context switch bound of 2, so its search is not guaranteed to be complete. SCALE was run with the not seen before and super step partial order reduction filters enabled. There are no cycles in the example, so there is no need to use the local cycle detection filter. In SCALE* we have also disabled the super step partial order reduction filter. Inspect and SCALE were run on 32bit Ubuntu Linux in a virtual machine on top of Mac OS X on a MacBook Pro with an Intel Core 2 Duo 2.4GHz CPU. CHESS was run on 32bit Windows XP in a virtual machine on the same MacBook



(a) number of runs

(b) verification time

Figure 6.4: Verification results for the *indexer* example

Pro. The results are also presented graphically in Figure 6.4.

Inspect uses the algorithm presented in [7] and so it manages to infer that a single run of the example is enough. CHESS can detect when the same state has been previously visited and so it is able to reduce the number of explored transitions, but not the number of explored states. As a consequence, it requires a lot more runs, even when context switch bounding is used. SCALE, using Super Step Partial Order Reduction, is able to infer that the threads are independent and can be considered as having just one super step, but is unable to deduce that executing any permutation of these threads has the same effect. We are currently investigating ways to include the Dynamic Partial Order Reduction algorithm presented in [7] in our framework and combine it with the rest of the reductions.

SCALE uses more memory than both Inspect and CHESS. While Inspect is stateless, it only keeps a stack of states for the current program run and does not store previously visited states, both SCALE and CHESS keep track of previously visited states. Because SCALE uses state filters it has to store more information for each state.

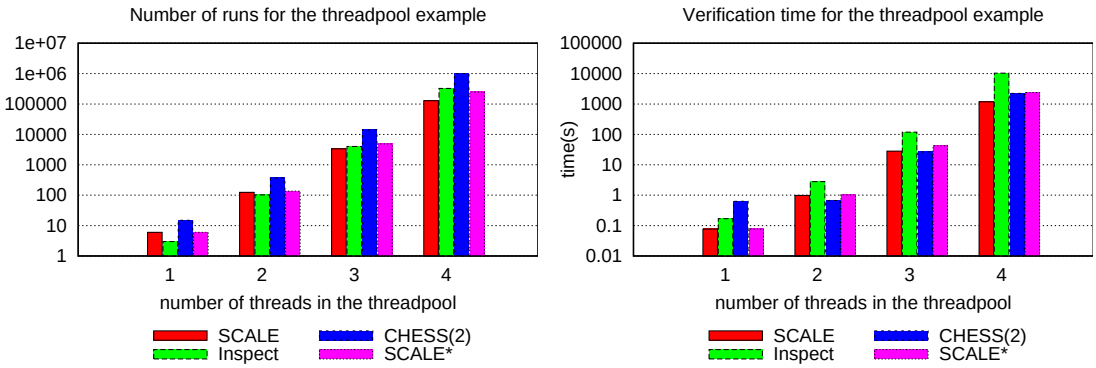
From Table 6.1 we can see that CHESS runs more tests per second than both Inspect and SCALE. The reason is that tests in CHESS are not separate programs, they are test cases run in the same process as the CHESS scheduler. Because consecutive runs of a test case happen in the same address space, state might be carried from one run to the other. The user must clean up everything at the end of the test case to ensure that any further runs have the same initial conditions. SCALE and Inspect run the scheduler and the verified program in two different processes, and use IPC mechanisms to communicate between them. While this approach adds communication overhead, it has the advantage that no cleanup code is necessary. The choice of IPC mechanism influences the performance, and for SCALE, we might get some improvement by switching from named pipes to shared memory.

6.3 Thread Pool

We have implemented a thread pool and verified it for absence of deadlock. The main thread creates *Size* pool threads, enqueues a job and then signals the thread pool to stop. There are many possible scenarios: the job can be run by any of the threads or it can be ignored if the pool threads don't get a chance to run before the main thread decides to stop them. We have implemented two versions: a Linux one, verified with SCALE and Inspect, and a Windows one, verified with CHESS. For the Linux implementation we used mutexes and condition variables to synchronize the threads. Conditions variables were not available in Windows, so we used semaphores and we tried to keep the implementation as close as possible to the Linux one. The source code of both versions is presented in Appendix A.3.

Tool	Size	Runs	Time(s)	Tool	Size	Runs	Time(s)
SCALE	1	6	0.077	SCALE	3	3347	28.143
Inspect	1	3	0.169	Inspect	3	4030	118.284
CHESS(2)	1	15	0.630	CHESS(2)	3	14024	26.593
SCALE*	1	6	0.077	SCALE*	3	4875	42.294
SCALE	2	124	0.980	SCALE	4	127766	1182.061
Inspect	2	102	2.788	Inspect	4	324772	10356.882
CHESS(2)	2	367	0.672	CHESS(2)	4	998381	2230.953
SCALE*	2	136	1.020	SCALE*	4	246763	2370.004

Table 6.2: Verification results for the *thread pool* example



(a) number of runs

(b) verification time

Figure 6.5: Verification results for the *thread pool* example

Table 6.2 presents the results obtained for different thread pool sizes. CHESS was run with a context switch bound of 2, and so it does not guarantee the completeness of the search. SCALE has the not seen before and super step partial order reduction filters enabled. SCALE* has only the not seen before filter enabled. We can see that for larger thread pool sizes, 3 and 4 in the table, SCALE needs fewer runs to complete the verification, but this comes at the price of using more memory than both Inspect and CHESS. The same results are also presented in Figure 6.5.

6.4 Reusable Barrier

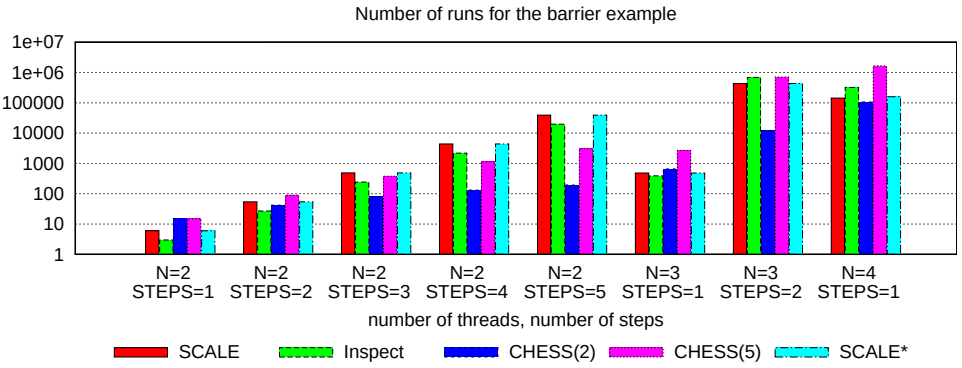
We have implemented a *reusable barrier* and verified it for various number of threads (N) and steps ($STEPS$). In each step each thread blocks at the barrier and waits for the other threads to reach the same point. When all the threads have reached the barrier, they all continue. This example turned out to be one in which the super step partial order reduction does not help much. We have implemented two versions of the barrier example, one for Linux and one for Windows. The Linux version uses mutexes and condition variables, while the Windows version uses critical sections and semaphores. The code is listed in Appendix A.4.

Tool	N	STEPS	Runs	Time(s)	Tool	N	STEPS	Runs	Time(s)
SCALE	2	1	6	0.068	SCALE	2	5	39365	430.832
Inspect	2	1	3	0.107	Inspect	2	5	19683	1196.423
CHESS(2)	2	1	15	0.470	CHESS(2)	2	5	195	0.375
CHESS(5)	2	1	15	0.470	CHESS(5)	2	5	3066	9.150
SCALE*	2	1	6	0.076	SCALE*	2	5	39365	435.793
SCALE	2	2	54	0.428	SCALE	3	1	481	3.603
Inspect	2	2	27	0.857	Inspect	3	1	390	12.792
CHESS(2)	2	2	42	0.125	CHESS(2)	3	1	658	1.218
CHESS(5)	2	2	90	0.188	CHESS(5)	3	1	2685	4.656
SCALE*	2	2	54	0.432	SCALE*	3	1	481	3.663
SCALE	2	3	486	4.576	SCALE	3	2	431894	4119.880
Inspect	2	3	243	12.806	Inspect	3	2	683100	34764.863
CHESS(2)	2	3	81	0.204	CHESS(2)	3	2	12182	21.641
CHESS(5)	2	3	378	0.609	CHESS(5)	3	2	712912	1276.906
SCALE*	2	3	486	4.467	SCALE*	3	2	431904	4132.043
SCALE	2	4	4374	43.542	SCALE	4	1	143639	1245.443
Inspect	2	4	2187	85.904	Inspect	4	1	321008	9562.527
CHESS(2)	2	4	132	0.282	CHESS(2)	4	1	107307	210.641
CHESS(5)	2	4	1140	1.906	CHESS(5)	4	1	1668069	3239.891
SCALE*	2	4	4374	44.073	SCALE*	4	1	158758	1466.807

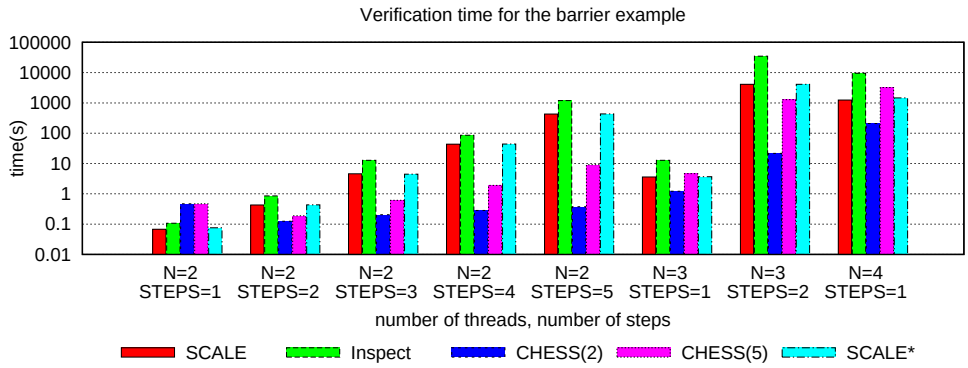
Table 6.3: Verification results for the *reusable barrier* example

In Table 6.3 SCALE has the not seen before and the super step partial order reduction filters enabled, while SCALE* has only the not seen before filter enabled. To observe the reduction power of context switch bounding, we have used CHESS with two context switch bounds: 2 and 5. From Table 6.3, we can see that an exploration with a context switch bound of 5 needs a lot more runs than one with a context switch bound of 2. Empirically it was observed that most bugs are found with a low context switch bound [17], so it would be useful to add this reduction to SCALE. The same results are also presented in Figure 6.6.

We have reasons to believe that Inspect reduces the state space too much so the search is no longer complete. For the two threads and one iteration experiment, after creating the additional thread, Inspect always continues to run the main thread and never tries to schedule the new thread fist. The order in which the two threads are run is important since they have dependent transitions.



(a) number of runs



(b) verification time

Figure 6.6: Verification results for the *reusable barrier* example

In all the runs checked by Inspect, the second thread is the last one that arrives at the barrier.

6.5 Other Interesting Examples

We have run SCALE on some buggy programs that pass either Inspect's or CHESS's checks and we were able to detect the errors. One such example is the producer-consumer problem in which we assert that when the consumer tries to take an element from the buffer, the buffer is empty and the consumer must wait for the producer to put an element in the buffer. It is easy to see that this assertion is violated if the producer runs first and manages to put an element in the buffer before the consumer tries to get an element from the buffer. Both CHESS and Inspect do not detect the assertion failure when the producer puts only one element in the buffer and the consumer takes only one element from the buffer. This only happens when the consumer is run in the main thread and the producer is run in another thread, if run the other way, the consumer in the new thread and the producer in the main thread, the error is detected by both CHESS and Inspect. The problem is that the producer does not get a chance to run immediately after it is created, instead the consumer is always scheduled first. The source code for this example (both the Linux and Windows versions)

is presented in Appendix A.5.

We can construct even simpler examples where the other tools miss some assertion failures. In Appendix A.6 we present such an example that has an execution leading to an assertion violation that is found by SCALE, but missed by Inspect. The lock is used to ensure that there are no race conditions.

The assertion failure happens when *thread_ b* runs before *thread_ a*. This is very likely to occur in practice and it can be easily exposed by compiling the program and running the resulting executable a few times.

Chapter 7

Conclusions and Future Directions

In this thesis we have presented the design and the implementation of the SCALE systematic software testing tool including its two major state space reduction algorithms: local execution cycle detection and super step partial order reduction.

We have applied SCALE to a C implementation of a non-blocking queuing algorithm and we were able to find a previously unknown bug. Using the algorithms presented in this thesis, the state space was reduced considerably, and this allowed us to find the error faster by taking fewer steps and exploring fewer distinct states. The local execution cycle detection allowed us to find the error even when we didn't bound the depth of the search.

We have also applied SCALE to other applications and compared the results to those obtained by two other tools: CHESS and Inspect. In some cases SCALE's algorithms prove to be useful in reducing the number of runs necessary to verify the applications. There are also cases where the partial order reduction is not that helpful. One way to improve SCALE's performance is to reorganize the way states and the information associated with the state filters are stored so that they would take less space. One other way to improve SCALE's performance is to use shared memory instead of named pipes as the IPC mechanism.

For large applications, SCALE's backtracking mechanism, restarting the program from the initial state and following the same scheduling decisions, proves to be too expensive. Revisiting a state that is deep down in the state space graph, requires the execution of the application again up to that point. SCALE would benefit greatly from a check-pointing mechanism that would allow us to restart the application, not from the initial state, but from a closer ancestor state.

SCALE's design allows us to easily add, enable and disable various reduction algorithms. For the future, we plan to add even more state space reductions. Preemptive context switch bounding is a promising reduction, and it appears that it would integrate well with the already existing ones. Dynamic partial order reduction is another algorithm that might prove to be useful. We also plan to add support for a scripting language that would allow users to write and test their own reduction and exploration algorithms.

As we have presented in this thesis, SCALE decouples the reduction algorithms from the exploration algorithms. So far we have been using only depth first search as our exploration algorithm, but we plan to implement and try others too. It also seems that it would be easy to adapt SCALE for distributed exploration and verification [12].

SCALE only deals with the non determinism introduced by thread scheduling. For the future we plan to also handle non determinism caused by user input and system calls.

SCALE was implemented to work on Unix/Linux applications that use the pthread API, but we see no reason why it could not be ported to other platforms, and be extended to support other multi-threading models.

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Appendix A

A.1 Non-Blocking Queue Source Code

```
#include <stdio.h>
#include <pthread.h>
#include <assert.h>

#define N 10

struct element {
  int *val;
  unsigned int ref;
};

pthread_mutex_t cas_lock;
struct element Q[N];
unsigned int FRONT = 0;
unsigned int REAR = 0;

int CAS(struct element *shared, struct element *old, struct element *new) {
  int result = 0;
  pthread_mutex_lock (&cas_lock);
  if (shared->val == old->val && shared->ref == old->ref) {
    shared->val = new->val;
    shared->ref = new->ref;
    result = 1;
  } else {
    result = 0;
  }
  pthread_mutex_unlock (&cas_lock);
  return result;
}

int CASi (unsigned int *shared, unsigned int old, unsigned int new) {
  int result = 0;
  pthread_mutex_lock (&cas_lock);
  if (*shared == old) {
    *shared = new;
    result = 1;
  } else {
    result = 0;
  }
```

```
  pthread_mutex_unlock (&cas_lock);
  return result;
}

void enqueue (int *item) {
  unsigned int rear = 0;
  struct element x, tmp;
enq_try_again:
  rear = REAR;
  x.val = Q[rear % N].val;
  x.ref = Q[rear % N].ref;
  if (rear != REAR) {
    goto enq_try_again;
  }
  if (rear == FRONT + N) {
    goto enq_try_again;
  }
  if (x.val == NULL) {
    tmp.val = item;
    tmp.ref = x.ref + 1;
    if (CAS (&Q[rear%N], &x, &tmp)) {
      CASi (&REAR, rear, rear + 1);
      return;
    }
  } else {
    CASi(&REAR, rear, rear + 1);
  }
  goto enq_try_again;
}

int * dequeue () {
  unsigned int front = 0;
  struct element x, tmp;
deq_try_again:
  front = FRONT;
  x.val = Q[front % N].val;
  x.ref = Q[front % N].ref;
  if (front != FRONT) {
    goto deq_try_again;
  }
  if (front == REAR) {
    goto deq_try_again;
  }
  if (x.val != NULL) {
    tmp.val = NULL;
    tmp.ref = x.ref + 1;
    if (CAS(&Q[front%N], &x, &tmp)) {
      CASi(&FRONT, front, front + 1);
      return x.val;
    }
  } else {
    CASi(&FRONT, front, front + 1);
  }
  goto deq_try_again;
```

```
}

void * producer_main (void *arg) {
  static int items[N];
  int i = 0;
  for (i = 0; i < N; i++) {
    items[i] = i;
    enqueue (&items[i]);
  }
  return NULL;
}

void * consumer_main (void *arg) {
  int *item = NULL;
  int i = 0;
  for (i = 0; i < N; i++) {
    item = dequeue ();
    assert (*item == i);
  }
  return NULL;
}

int main (int argc, char **argv) {
  pthread_t tid;
  pthread_mutex_init (&cas_lock, NULL);
  pthread_create (&tid, NULL, producer_main, NULL);
  consumer_main (NULL);
  pthread_join (tid, NULL);
  pthread_mutex_destroy (&cas_lock);
  return 0;
}
```

A.2 Indexer Source Code

A.2.1 Linux Version

```
#include <pthread.h>

#define SIZE  128
#define MAX     4
#define N       8

int table[SIZE];
pthread_mutex_t cas_mutex[SIZE];
pthread_t tids[N-1];

int cas (int *tab, int h, int val, int new_val) {
  int result = 0;
  pthread_mutex_lock(&cas_mutex[h]);
  if (tab[h] == val) {
    tab[h] = new_val;
    result = 1;
  }
  pthread_mutex_unlock(&cas_mutex[h]);
```

```
  return result;
}

void thread_routine (int tid) {
  int m = 0, w, h;
  while(1){
    if ( m < MAX ) {
      w = (++m) * 11 + tid;
    }
    else {
      break;
    }
    h = (w * 7) % SIZE;
    while ( cas(table, h, 0, w) == 0) {
      h = (h+1) % SIZE;
    }
  }
}

void * thread_main (void *arg) {
  thread_routine ((int)(unsigned long)arg);
  return NULL;
}

int main (int argc, char **argv) {
  int i;
  for (i = 0; i < SIZE; i++) {
    pthread_mutex_init(&cas_mutex[i], NULL);
  }
  for (i = 0; i < N-1; i++) {
    pthread_create(&tids[i], NULL,  thread_main,
                   (void*)(unsigned long)(i));
  }
  thread_routine(N-1);
  for (i = 0; i < N-1; i++) {
    pthread_join(tids[i], NULL);
  }
  for (i = 0; i < SIZE; i++) {
    pthread_mutex_destroy(&cas_mutex[i]);
  }
  return 0;
}
```

A.2.2 Windows Version

```
/* indexer.cpp */

#include <windows.h>
#include <iostream>
#include <assert.h>

#define SIZE 128
#define MAX 4
#define N 7
```

```
int table[SIZE];
CRITICAL_SECTION cas_mutex[SIZE];
HANDLE threads[N-1];
DWORD tids[N-1];

int cas (int *tab, int h, int val, int new_val) {
  int result = 0;
  EnterCriticalSection (&cas_mutex[h]);
  if ( tab[h] == val ) {
    tab[h] = new_val;
    result = 1;
  }
  LeaveCriticalSection (&cas_mutex[h]);
  return result;
}

void thread_routine (int tid) {
  int m = 0, w, h;
  while(1) {
    if ( m < MAX ) {
      w = (++m) * 11 + tid;
    }
    else {
      break;
    }
    h = (w * 7) % SIZE;
    while ( cas(table, h, 0, w) == 0) {
      h = (h+1) % SIZE;
    }
  }
}

DWORD WINAPI thread_main (LPVOID arg)
{
  thread_routine ((int)(unsigned long)arg);
  return 0;
}

extern "C"
__declspec(dllexport) int ChessTestRun () {
  int i = 0;
  for (i = 0; i < SIZE; i++) {
    table[i] = 0;
    InitializeCriticalSection (&cas_mutex[i]);
  }
  for (i = 0; i < N-1; i++) {
    threads[i] = CreateThread (NULL, 0, thread_main,
                               (LPVOID)(unsigned long)i, 0, &tids[i]);
  }
  thread_routine (N-1);
  for (i = 0; i < N-1; i++) {
    WaitForSingleObject (threads[i], INFINITE);
    CloseHandle (threads[i]);
```

```
  }
  for (i = 0; i < SIZE; i++) {
    DeleteCriticalSection (&cas_mutex[i]);
  }
  return 0;
}
```

A.3 Thread Pool Source Code

A.3.1 Linux Version

```
#include <pthread.h>
#include <assert.h>
#include <stdio.h>

#define N 4

typedef void (*job)(void);

struct node {
  job job;
  struct node *next;
};

struct queue {
  struct node *head;
  struct node *tail;
};

struct queue jobs;
pthread_mutex_t lock;
pthread_cond_t wakeup_cond;
int exiting = 0;
int idle = 0;

job dequeue_job () {
  struct node *v = NULL;
  job job;
  v = jobs.head;
  job = v->job;
  jobs.head = v->next;
  free (v);
  if (jobs.head == NULL) {
    jobs.tail = NULL;
  }
  return job;
}

void enqueue_job (job job) {
  struct node *n = NULL;
  pthread_mutex_lock (&lock);
  n = (struct node *) malloc (sizeof (struct node));
  assert (n != NULL);
  n->job = job;
```

```
  n->next = NULL;
  if (jobs.tail == NULL) {
    assert (jobs.head == NULL);
    jobs.head = n;
  }
  else {
    jobs.tail->next = n;
  }
  jobs.tail = n;
  if (idle > 0) {
    pthread_cond_signal (&wakeup_cond);
  }
  pthread_mutex_unlock (&lock);
}

void * thread_main (void *arg) {
  job job;
  while (1) {
    pthread_mutex_lock (&lock);
    while ((jobs.head == NULL) && !exiting) {
      idle++;
      pthread_cond_wait (&wakeup_cond, &lock);
      idle--;
    }
    if (exiting) {
      pthread_mutex_unlock (&lock);
      break;
    } else {
      job = dequeue_job ();
      pthread_mutex_unlock (&lock);
      job ();
    }
  }
  return NULL;
}

void
print_hello () {
  printf ("%s", "Hello!\n");
}

int main (int argc, char **argv) {
  pthread_t threads[N];
  int i = 0;
  pthread_mutex_init (&lock, NULL);
  pthread_cond_init (&wakeup_cond, NULL);
  for (i = 0; i < N; i++) {
    pthread_create (&threads[i], NULL, thread_main, NULL);
  }
  enqueue_job (print_hello);
  pthread_mutex_lock (&lock);
  exiting = 1;
  pthread_cond_broadcast (&wakeup_cond);
  pthread_mutex_unlock (&lock);
```

```
  for (i = 0; i < N; i++) {
    pthread_join (threads[i], NULL);
  }
  pthread_mutex_destroy (&lock);
  pthread_cond_destroy (&wakeup_cond);
  return 0;
}
```

A.3.2 Windows Version

```
#include <windows.h>
#include <iostream>
#include <stdio.h>
#include <assert.h>

#define N 4

typedef void (*job)(void);

struct node {
  job job;
  struct node *next;
};

struct queue {
  struct node *head;
  struct node *tail;
};

struct queue jobs= {NULL, NULL};
CRITICAL_SECTION lock;
HANDLE wakeup_sem;
int exiting = 0;
int idle = 0;

job dequeue_job () {
  struct node *v = NULL;
  job job;
  v = jobs.head;
  job = v->job;
  jobs.head = v->next;
  free (v);
  if (jobs.head == NULL) {
    jobs.tail = NULL;
  }
  return job;
}

void enqueue_job (job job) {
  struct node *n = NULL;
  EnterCriticalSection (&lock);
  n = (struct node *) malloc (sizeof (struct node));
  assert (n != NULL);
  n->job = job;
```

```
  n->next = NULL;
  if (jobs.tail == NULL) {
    assert (jobs.head == NULL);
    jobs.head = n;
  } else {
    jobs.tail->next = n;
  }
  jobs.tail = n;
  if (idle > 0) {
    ReleaseSemaphore (wakeup_sem, 1, NULL);
  }
  LeaveCriticalSection (&lock);
}

DWORD WINAPI thread_main (LPVOID param)
{
  job job;
  while (1) {
    EnterCriticalSection (&lock);
    while ((jobs.head == NULL) && !exiting) {
      idle++;
      LeaveCriticalSection (&lock);
      WaitForSingleObject (wakeup_sem, INFINITE);
      EnterCriticalSection (&lock);
      idle--;
    }
    if (exiting) {
      LeaveCriticalSection (&lock);
      break;
    } else {
      job = dequeue_job ();
      LeaveCriticalSection (&lock);
      job ();
    }
  }
  return 0;
}

void print_hello () {
  printf ("%s", "Hello!\n");
}

using namespace std;

void cleanup () {
  node *v = NULL;
  exiting = 0;
  idle = 0;
  while (jobs.head != NULL) {
    v = jobs.head;
    jobs.head = v->next;
    free (v);
  }
  jobs.head = NULL;
```

```
  jobs.tail = NULL;
}

extern "C"
__declspec(dllexport) int ChessTestRun() {
  DWORD tid[N];
  HANDLE hThread[N];
  unsigned int i = 0;
  cleanup ();
  InitializeCriticalSection (&lock);
  wakeup_sem = CreateSemaphore (NULL, 0, N+1, NULL);
   for (i = 0; i < N; i++) {
    hThread[i] = CreateThread (NULL, 0, thread_main,
                               (LPVOID)(unsigned long)i, 0, &tid[i]);
  }
  enqueue_job (print_hello);
  EnterCriticalSection (&lock);
  exiting = 1;
  ReleaseSemaphore (wakeup_sem, idle, NULL);
  LeaveCriticalSection (&lock);
  for (i = 0; i < N; i++) {
    WaitForSingleObject (hThread[i], INFINITE);
    CloseHandle(hThread[i]);
  }
  DeleteCriticalSection (&lock);
  CloseHandle (wakeup_sem);
  return 0;
}
```

A.4 Barrier Source Code

A.4.1 Linux Version

```
#include <pthread.h>

#define N 4
#define STEPS 1

pthread_mutex_t lock;
pthread_cond_t cond;
int count = 0;
int phase = 0;

void phase_0 () {
  pthread_mutex_lock (&lock);
  count++;
  if (count == N) {
    phase = 1;
    pthread_cond_broadcast (&cond);
  } else {
    while (phase == 0) {
      pthread_cond_wait (&cond, &lock);
    }
  }
```

```
  pthread_mutex_unlock (&lock);
}

void phase_1 () {
  pthread_mutex_lock (&lock);
  count--;
  if (count == 0) {
    phase = 0;
    pthread_cond_broadcast (&cond);
  } else {
    while (phase == 1) {
      pthread_cond_wait (&cond, &lock);
    }
  }
  pthread_mutex_unlock (&lock);
}

void barrier () {
  phase_0 ();
  phase_1 ();
}

void process (void *arg) {
  int i = 0;
  for (i = 0; i < STEPS; i++) {
    barrier ();
  }
}

void * thread_main (void *arg) {
  process (arg);
  return NULL;
}

int main (int argc, char **argv) {
  pthread_t threads[N-1];
  int i = 0;
  pthread_mutex_init (&lock, NULL);
  pthread_cond_init (&cond, NULL);
  for (i = 0; i < N-1; i++) {
    pthread_create (&threads[i], NULL, thread_main, NULL);
  }
  process (NULL);
  for (i = 0; i < N-1; i++) {
    pthread_join (threads[i], NULL);
  }
  pthread_mutex_destroy (&lock);
  pthread_cond_destroy (&cond);
  return 0;
}
```

A.4.2 Windows Version

```
#include <windows.h>
```

```
#include <iostream>
#include <assert.h>

#define N 3
#define STEPS 2

CRITICAL_SECTION lock;
HANDLE sem1, sem2;
unsigned int count = 0;

void phase_1 () {
  int signaller = 0;
  EnterCriticalSection (&lock);
  count++;
  if (count == N) {
    signaller = 1;
    ReleaseSemaphore (sem1, N-1, NULL);
  }
  LeaveCriticalSection (&lock);
  if (!signaller) {
    WaitForSingleObject (sem1, INFINITE);
  }
}

void phase_2 () {
  int signaller = 0;
  EnterCriticalSection (&lock);
  count--;
  if (count == 0) {
    signaller = 1;
    ReleaseSemaphore (sem2, N-1, NULL);
  }
  LeaveCriticalSection (&lock);
  if (!signaller) {
    WaitForSingleObject (sem2, INFINITE);
  }
}

void barrier () {
  phase_1 ();
  phase_2 ();
}
DWORD WINAPI thread_main(LPVOID param) {
  int i = 0;
  for (i = 0; i < STEPS; i++) {
    barrier ();
  }
  return 0;
}

using namespace std;

extern "C"
__declspec(dllexport) int ChessTestRun() {
```

```
  DWORD tid[N-1];
  HANDLE hThread[N-1];
  unsigned int i = 0;
  count = 0;
  InitializeCriticalSection (&lock);
  sem1 = CreateSemaphore (NULL, 0, N-1, NULL);
  sem2 = CreateSemaphore (NULL, 0, N-1, NULL);
  for (i = 0; i < N-1; i++) {
    hThread[i] = CreateThread (NULL, 0, thread_main,
                               (LPVOID)(unsigned long)i, 0, &tid[i]);
  }
  thread_main ((LPVOID)(unsigned long)(N-1));
  for (i = 0; i < N-1; i++) {
    WaitForSingleObject (hThread[i], INFINITE);
    CloseHandle(hThread[i]);
  }
  DeleteCriticalSection (&lock);
  CloseHandle (sem1);
  CloseHandle (sem2);
  return 0;
}
```

A.5 Producer-Consumer Source Code

A.5.1 Linux Version

```
#include <assert.h>
#include <pthread.h>

#define SIZE 2
#define N 1

struct semaphore {
  pthread_mutex_t lock;
  pthread_cond_t cond;
  int value;
  int flag;
};

void semaphore_init (struct semaphore *sem, int value) {
  pthread_mutex_init (&sem->lock, NULL);
  pthread_cond_init (&sem->cond, NULL);
  sem->value = value;
  sem->flag = 0;
}

void semaphore_destroy (struct semaphore *sem) {
  pthread_mutex_destroy (&sem->lock);
  pthread_cond_destroy (&sem->cond);
}

void semaphore_wait (struct semaphore *sem) {
  int tvalue;
  pthread_mutex_lock (&sem->lock);
```

```
  tvalue = sem->value;
  tvalue--;
  sem->value = tvalue;
  if (tvalue < 0) {
    while (sem->flag == 0) {
      pthread_cond_wait (&sem->cond, &sem->lock);
    }
    sem->flag = 0;
  }
  pthread_mutex_unlock (&sem->lock);
}

void semaphore_post (struct semaphore *sem) {
  int tvalue;
  pthread_mutex_lock (&sem->lock);
  tvalue = sem->value;
  tvalue++;
  sem->value = tvalue;
  if (tvalue <= 0) {
    sem->flag = 1;
    pthread_cond_signal (&sem->cond);
  }
  pthread_mutex_unlock (&sem->lock);
}

pthread_mutex_t mutex;
struct semaphore items, spaces;
unsigned int fill = 0;
unsigned char buffer[SIZE];


void * producer (void *arg) {
  int i;
  unsigned char item;
  for (i = 0; i < N; i++) {
    item = (unsigned char)(i%256);
    semaphore_wait (&spaces);
    pthread_mutex_lock (&mutex);
    buffer[i%SIZE] = item;
    fill++;
    pthread_mutex_unlock (&mutex);
    semaphore_post (&items);
  }
  return NULL;
}

void * consumer (void *arg) {
  int i = 0;
  unsigned char item;
  assert (fill == 0);
  for (i = 0; i < N; i++) {
    semaphore_wait (&items);
    pthread_mutex_lock (&mutex);
    item = buffer[i%SIZE];
```

```
    fill--;
    assert (item == (unsigned char)(i%256));
    pthread_mutex_unlock (&mutex);
    semaphore_post (&spaces);
  }
  return NULL;
}

int main (int argc, char **argv) {
  pthread_t t;
  pthread_mutex_init (&mutex, NULL);
  semaphore_init (&items, 0);
  semaphore_init (&spaces, SIZE);
  pthread_create (&t, NULL, producer, NULL);
  consumer (NULL);
  pthread_join (t, NULL);
  pthread_mutex_destroy (&mutex);
  semaphore_destroy (&items);
  semaphore_destroy (&spaces);
  return 0;
}
```

A.5.2 Windows Version

```
#include <windows.h>
#include <iostream>
#include <assert.h>

#define SIZE 2
#define N 1

CRITICAL_SECTION mutex;
HANDLE items, spaces;
unsigned int fill = 0;
unsigned char buffer[SIZE];

DWORD WINAPI producer (LPVOID param) {
  int i;
  unsigned char item;
  for (i = 0; i < N; i++) {
    item = (unsigned char)(i%256);
    WaitForSingleObject (spaces, INFINITE);
    EnterCriticalSection (&mutex);
    buffer[i%SIZE] = item;
    fill++;
    LeaveCriticalSection (&mutex);
    ReleaseSemaphore (items, 1, NULL);
  }
  return 0;
}

DWORD WINAPI consumer(LPVOID param) {
  int i = 0;
  unsigned char item;
```

```
  for (i = 0; i < N; i++) {
    assert (fill == 0);
    WaitForSingleObject (items, INFINITE);
    EnterCriticalSection (&mutex);
    item = buffer[i%SIZE];
    fill--;
    assert (item == (unsigned char)(i%256));
    LeaveCriticalSection (&mutex);
    ReleaseSemaphore (spaces, 1, NULL);
  }
  return 0;
}

using namespace std;

extern "C"
__declspec(dllexport) int ChessTestRun() {
  DWORD tp;
  HANDLE prod;
  unsigned int i = 0;
  fill = 0;
  InitializeCriticalSection(&mutex);
  items = CreateSemaphore (NULL, 0, SIZE, NULL);
  spaces = CreateSemaphore (NULL, SIZE, SIZE, NULL);
  prod = CreateThread (NULL, 0, producer, NULL, 0, &tp);
  consumer (NULL);
  WaitForSingleObject (prod, INFINITE);
  DeleteCriticalSection(&mutex);
  CloseHandle (prod);
  CloseHandle (items);
  CloseHandle (spaces);
  return 0;
}
```

A.6 Bug Inspect

```
#include <pthread.h>
#include <assert.h>

int x = 2;
pthread_mutex_t lock;

void * thread_a (void *arg) {
  int tmp;
  pthread_mutex_lock (&lock);
  tmp = x;
  pthread_mutex_unlock (&lock);
  tmp++;
  pthread_mutex_lock (&lock);
  x = tmp;
  pthread_mutex_unlock (&lock);
  return NULL;
}
```

```
void * thread_b (void *arg) {
  int tmp;
  pthread_mutex_lock (&lock);
  tmp = x;
  pthread_mutex_unlock (&lock);
  tmp *= 2;
  pthread_mutex_lock (&lock);
  x = tmp;
  pthread_mutex_unlock (&lock);
  return NULL;
}

int main (int argc, char **argv) {
  pthread_t p;
  pthread_mutex_init (&lock, NULL);
  pthread_create (&p, NULL,
                  thread_b, NULL);
  thread_a (NULL);
  pthread_join (p, NULL);
  pthread_mutex_destroy (&lock);
  assert (x != 5);
  return 0;
}
```